Magnetohydrodynyamic Flow for a Viscoclastic Fluid with the Generalized Oldroyd-B Model with Fractional Derivative

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Abstract

This paper deals with the Magnetohydrodynyamic (Mill)) flow for a viscoclastic fluid of the generalized Oldroyd-B model. The fractional calculus approach is used to establish the constitutive relationship of the non-Newtonian fluid model. Exact analytic solutions for the velocity and shear stress fields in terms of the Fox H-function are obtained by using discrete Laplace transform. The effect of different parameter that controlled the motion and shear stress equations are studied through plotting using the MATHEMATICA-8 software.

Keywords: MHD, viscoelastic fluid, generalized Oldroyd-B, fractional derivative.

جريان ممغنط لمائع لزج–مطاطي من النمط أولدرويد–بي ذو المشتقات الكسرية

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الخلاصة

هذا البحث يتناول جريان ممغنط لمائع لزج–مطاطي من النمط أولدرويد– بي. التفاضل الكسري قد أستخدم لكتابة المعادلات المحكمة لهذا المائع اللانيوتيني. الحلول المضبوطة لكل من حقل السرعة وإجهاد القص قد تم الحصول عليها باستخدام تحويلات لابلاس وبدلالة دالة H–فوكس. وأخيراً تمت دراسة تأثير كل من المعلمات التي تحكم كل من حقل السرعة وإجهاد القص من رسم ذلك بإستخدام البرنامج الجاهز ماثيماتيكا–8.

Introduction

In technological applications, it has generally known that non-Newtonian fluids (such as molten plastic, paints, blood and other similar entities) are more appropriate and suitable that Newtonian fluids. This is due to their wide area of their applications, such as exotic lubricants, food stuffs, colloidal and suspension solution, slurry fuels and many others. Because of the complex of the non-Newtonian fluids there is no model which can alone describe them all. Therefore, several constitutive equations for non-Newtonian fluid have been proposed. One of them, the Oldroyd-B fluid viscoelastic fluid model which can predict stress relaxation, has much attention. Fractional derivatives have been found to be quite flexible in describing viscoelastic behavior, [1]. In general, the constitutive equations for generalized non-Newtonian fluids arc modified from the well known models by replacing the time derivative of an integer order with the so-called Riemann-Liouville fractional calculus operator, [2-3].

Haitao and Xu [4] investigated the Stokes problem for viscoelastic fluid with the generalized Oldroyd-B fluid model. Khan et al. [5] discussed some accelerated flows for generalized Oldroyd-B fluid. Hyder et al. [6] investigated the generalized Oldroyd-B fluid model with the fractional calculus approach is used. They considered two types of flow namely, (i) flow due to impulsive motion in present of constant pressure gradient and (ii) flow induced by an impulsive pressure gradient, obtained

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an exact analytic solution for the velocity and stress fields in terms of Fox H-function. Also, they analyzed the influence of various parameters of interest on the velocity and shear stress through several graphs, a comparison between Oldroyd-B fluid and generalized Oldroyd-B fluid is also given.
In this paper, the effect of magnatichyrodynamic (MHD) for a viscoelastic fluid with the generalized Oldroyd-B model with fractional derivative is studied, also, the effect of different parameter that controlled the motion and shear stress equations are analyzed through plotting using the MATHEMATICA-8 software.

Formulation of the Problem

The fundamental equations governing the unsteady motion of an incompressible fluid are:

$$div\ V = 0, \tag{1}$$

$$\rho \frac{dV}{dt} = div\ T + \rho b \tag{2}$$

where V is the velocity field, ρ is the density, T is the Cauchy stress tensor, σ is the electrical conductivity of the fluid, b is the body force and $\frac{d}{dt}$ the material time derivative.

The Cauchy stress tensor T for a generalized Oldroyd-B fluid [4,7] is:

$$T = -pI + S,$$

$$\left(1+\lambda^{\alpha}\frac{D^{\alpha}}{Dt^{\alpha}}\right)S = \mu\left(1+\theta^{\alpha}\frac{D^{\beta}}{Dt^{\beta}}\right)A_1 \tag{3}$$

In which p is the pressure, I is the identity tensor, μ the dynamic viscosity, S the extra stress tensor, λ and θ are relaxation and retardation tines respectively, α and β are fractional calculus parameter such that $0 \le \alpha \le \beta \le 1$, and first Rivlin-Ericksen tensor is given by:

$$A_1 = L + L^T,\ L = grad\ V \tag{4}$$

where T denoted the matrix transpose,

$$\frac{D^{\alpha}S}{Dt^{\alpha}} = D_t^{\alpha} S + (V.\nabla)S - LS - LS^T, \tag{5}$$

$$\frac{D^{\beta}A_1}{Dt^{\beta}} = D_t^{\beta} A_1 + (V.\nabla) A_1 - LS - L A_1^T\ , \tag{6}$$

In which D_t^{α} and D_t^{β} are the fractional differentiation operators of order α and β with respect to t, respectively and may be defined as:

$$D_t^p [f(t)] = \frac{1}{\Gamma(1-p)}\frac{d}{dt}\int_0^t \frac{f(\tau)}{(1-\tau)}d\tau\ ,\ 0 \le p \le 1$$

where $\Gamma(.)$ in the Gamma function.

For unidirectional flow, we consider the velocity and the stress of the form:

$$V = u(y,t)i,\ S = S(y,t) \tag{7}$$

where u is the velocity component in the x-direction and i is the unit vector in the x-direction.

Now, we have:

$$L = \begin{bmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{bmatrix},\ L^T = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix},\ S = \begin{bmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{bmatrix}$$

Since:

$$V = u(y,t)i,\ S = S(y,t)$$

For the problem under consideration, equation (2) takes the form:

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \beta_0^2 u \tag{8}$$

Also:

$S(y,0) = \partial_t S(y,0) = 0,\ S_{yy} = S_{zz} = S_{xz} = S_{yz} = 0,\ S_{xy} = S_{yx}$

[Here S_{yz} means the tangential stress on a phase whose normal is y and acting in the z direction, and so on]. And:

$$L = \begin{bmatrix} 0 & \frac{\partial u}{\partial y} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ L^T = \begin{bmatrix} 0 & 0 & 0 \\ \frac{\partial u}{\partial y} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ A = \begin{bmatrix} 0 & \frac{\partial u}{\partial y} & 0 \\ \frac{\partial u}{\partial y} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ LS = \begin{bmatrix} \frac{\partial u}{\partial y} S_{yx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$LS^T = \begin{bmatrix} \frac{\partial u}{\partial y} S_{xy} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ LA = \begin{bmatrix} \left(\frac{\partial u}{\partial y}\right)^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ AL^T = \begin{bmatrix} \left(\frac{\partial u}{\partial y}\right)^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(V.\nabla)S = u\frac{\partial}{\partial x}\begin{bmatrix} S_{xx} & S_{xy} & 0 \\ S_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0,\ (V.\ \nabla)A = u\frac{\partial}{\partial x}\begin{bmatrix} 0 & \frac{\partial u}{\partial y} & 0 \\ \frac{\partial u}{\partial y} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Since

$$\left(1 + \lambda^\alpha \frac{D^\alpha}{Dt^\alpha}\right) S = \mu\left(1 + \theta^\alpha \frac{D^\beta}{Dt^\beta}\right) A_1$$

Thus, using the above expressions, we obtain:

$$S + \lambda^\alpha(D_t^\alpha S + (\vec{v}\,.\nabla)S - LS - SL^T) = \mu(A + \theta^\beta(D_t^\beta A + (\vec{v}\,.\nabla)A - LA - AL^T)) \qquad (9)$$

The second term in the L.H.S is given by:

$$D_t^\alpha S + (\vec{v}\,.\nabla)S - LS - SL^T$$

$$= D_t^\alpha \begin{bmatrix} S_{xx} & S_{xy} & 0 \\ S_{yx} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + 0 - \begin{bmatrix} \frac{\partial u}{\partial y} S_{xy} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} \frac{\partial u}{\partial y} S_{xy} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} D_t^\alpha S_{xx} - 2\frac{\partial u}{\partial y} S_{xy} & D_t^\alpha S_{xy} & 0 \\ D_t^\alpha S_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (10)$$

The R.H.S term is:

$$D_t^\beta A + (\vec{v}\,.\nabla)A - LA - AL^T$$

$$= D_t^{\beta}\begin{bmatrix} 0 & \frac{\partial u}{\partial y} & 0 \\ \frac{\partial u}{\partial y} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + 0 - \begin{bmatrix} \left(\frac{\partial u}{\partial y}\right)^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} \left(\frac{\partial u}{\partial y}\right)^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -2\left(\frac{\partial u}{\partial y}\right)^2 & D_t^{\beta}\frac{\partial u}{\partial y} & 0 \\ D_t^{\beta}\frac{\partial u}{\partial y} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ....(11)$$

By using equation (10) and (11), equation (9) can he written as:

$$\begin{bmatrix} S_{xx} & S_{xy} & 0 \\ S_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \lambda^{\alpha}\begin{bmatrix} D_t^{\alpha}S_{xx} - 2\frac{\partial u}{\partial y}S_{xy} & D_t^{\alpha}S_{xy} & 0 \\ D_t^{\alpha}S_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \mu\left(\begin{bmatrix} 0 & \frac{\partial u}{\partial y} & 0 \\ \frac{\partial u}{\partial y} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \theta^{\beta}\begin{bmatrix} -2\left(\frac{\partial u}{\partial y}\right)^2 & D_t^{\beta}\frac{\partial u}{\partial y} & 0 \\ D_t^{\beta}\frac{\partial u}{\partial y} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right)$$

From the last expression, one can obtain:

$$(1 + \lambda^{\alpha} D_t^{\alpha})S_{xy} = \mu(1 + \theta^{\beta} D_t^{\beta})\frac{\partial u}{\partial y} \quad (12)$$

Eliminating S_{xy} between equations (8) and (12). we obtain:

$$\rho(1 + \lambda^{\alpha} D_t^{\alpha})\frac{\partial u}{\partial t} = -(1 + \lambda^{\alpha} D_t^{\alpha})\frac{\partial p}{\partial x} + \mu(1 + \theta^{\beta} D_t^{\beta})\frac{\partial^2 u}{\partial y^2} - \beta_0^2(1 + \theta^{\beta} D_t^{\beta})u \quad (13)$$

The governing equation, in the flow direction, is given by:

$$(1 + \lambda^{\alpha} D_t^{\alpha})\frac{\partial u}{\partial t} = -(1 + \lambda^{\alpha} D_t^{\alpha})\frac{\partial p}{\partial x} + v(1 + \theta^{\beta} D_t^{\beta})\frac{\partial^2 u}{\partial y^2} - \mathrm{M}(1 + \lambda^{\alpha} D_t^{\alpha})u \quad (14)$$

where $v = \frac{\mu}{\rho}$ is the kinematics v of the viscosity fluid and $M = \frac{\sigma B_0^2}{\rho}$ is the magnetic field parameter.

It's clear that if $M = 0$, we obtain the corresponding equation for on an incompressible Oldroyd-B fluid with fractional derivative as obtained by [6].

Calculation of the Velocity

Let us consider the flowing problem of the effect of MHD on an incompressible Oldroyd-B fluid with fractional derivative model occupies plane $y \geq 0$ and x-axis is chosen as plane wall We will assume that the system initially at rest and at $t = 0$ the fluid is suddenly set in motion due to a constant pressure gradient and by the motion of the plane wall. In this case the governing partial differential equation and the initial and the boundary conditions are of the form:

$$(1 + \lambda^{\alpha} D_t^{\alpha})\frac{\partial u}{\partial t} = -(1 + \lambda^{\alpha}\frac{t^{-\alpha}}{\Gamma(1-\alpha)})A + v(1 + \theta^{\beta} D_t^{\beta})\frac{\partial^2 u}{\partial y^2} - \mathrm{M}(1 + \lambda^{\alpha} D_t^{\alpha})u \quad (15)$$

where $A = (1/\rho)\dfrac{\partial p}{\partial x}$ is the constant pressure gradient.

$$u(y,0) = \frac{\partial u}{\partial t}(y,0) = 0,\ y \succ 0 \tag{16}$$

$$u(0,t) = U,\ t \succ 0 \tag{17}$$

$$u(y,t),\ \frac{\partial u}{\partial t}(y,t) \longrightarrow 0, \text{ as } y \longrightarrow 0 \tag{18}$$

To obtain an exact solution of the above initial value problem, the Laplace transform is used. Let $\bar{u}(y,s)$ be the Laplace transform of $u(y,t)$ defined by:

$$\bar{u}(y,s) = \int_0^\infty u(y,t)e^{-st}\,dt,\ s \succ 0$$

Taking the Laplace transform of equations (15-18), we arrive at:

$$\frac{d^2\bar{u}}{dy^2} - \frac{(s+M)(1+\lambda^\alpha s^\alpha)}{v(1+\theta^\beta s^\beta)}\bar{u} = \frac{(1+\lambda^\alpha s^\alpha)}{v(1+\theta^\beta s^\beta)} - \frac{A}{s} \tag{19}$$

$$\bar{u}(0,s) = \frac{U}{s} \tag{20}$$

$$\bar{u}(y,s), \frac{\partial \bar{u}}{\partial y}(y,s) \longrightarrow 0, \text{ as } y \longrightarrow \infty \tag{21}$$

This is ordinary differential equation of second order. The solution of equation (19) that satisfying the boundary conditions (20) and (21) is of the following form:

$$\bar{u} = \frac{U}{s} e^{-\left(\frac{(s+M)(1+\lambda^\alpha s^\alpha)}{v(1+\theta^\beta s^\beta)}\right)^{1/2} y} + \frac{A}{s(s+M)}\left(1 - e^{-\left(\frac{(s+M)(1+\lambda^\alpha s^\alpha)}{v(1+\theta^\beta s^\beta)}\right)^{1/2} y}\right) \tag{22}$$

In the last equation if we set M is equal to zero we covering the same problem but in the absent of the magnetic field as appears in [6] equation (20).

In order to avoid the complicated calculations of residues and contour integrals, we apply the discrete inverse Laplace transform [1] to get the velocity distribution. Now, writing equation (22) in series form as:

$$\bar{u} = \frac{U}{s} + U\sum_{k=1}^{\infty}\frac{(-y)^k M^{k/2}}{v^{k/2}k!}\sum_{m=0}^{\infty}\frac{M^{-k/2+m}(-1)^m\Gamma\left(m-\frac{k}{2}\right)}{m!\Gamma\left(-\frac{k}{2}\right)}\sum_{n=0}^{\infty}\frac{\lambda^{\alpha k/2-\alpha n}(-1)^n\Gamma\left(n-\frac{k}{2}\right)}{n!\Gamma\left(-\frac{k}{2}\right)}$$

$$\sum_{l=0}^{\infty}\frac{(-1)^l\Gamma\left(l+\frac{k}{2}\right)}{l!\Gamma\left(\frac{k}{2}\right)\theta^{\beta k/2+\beta l}}\frac{1}{s^{k/2(\beta-\alpha-1)+m+\alpha n+\beta l+1}} + A\sum_{k=1}^{\infty}\frac{(-y)^k M^{k/2-1}}{v^{k/2}k!}$$

$$\sum_{r=0}^{\infty}\frac{M^{k/2-1}(-M)^r s^{-r}\Gamma\left(r-\left(1-\frac{k}{2}\right)\right)}{r!\Gamma\left(\frac{k}{2}-1\right)}\sum_{n=0}^{\infty}\frac{\lambda^{\alpha k/2-\alpha n}(-1)^n\Gamma\left(n-\frac{k}{2}\right)}{n!\Gamma\left(-\frac{k}{2}\right)}$$

$$\sum_{l=0}^{\infty}\frac{(-1)^l\Gamma\left(l+\frac{k}{2}\right)}{l!\Gamma\left(\frac{k}{2}\right)\theta^{\beta k/2+\beta l}}\frac{1}{s^{k/2(\beta-\alpha-1)+r+\alpha n+\beta l+2}} \tag{23}$$

The application of the discrete inverse Laplace transform to equation (23) gives:

$$u = U + \sum_{k=1}^{\infty}\frac{(-y)^k M^{k/2-1}}{v^{k/2}k!}\sum_{m=0}^{\infty}\frac{M^{-k/2+m}(-1)^m\Gamma\left(m-\frac{k}{2}\right)}{m!\Gamma\left(-\frac{k}{2}\right)}\sum_{n=0}^{\infty}\frac{\lambda^{\alpha k/2-\alpha n}(-1)^n\Gamma\left(n-\frac{k}{2}\right)}{n!\Gamma\left(-\frac{k}{2}\right)}$$

$$\sum_{l=0}^{\infty}\frac{(-1)^l\Gamma\left(l+\frac{k}{2}\right)}{l!\Gamma\left(\frac{k}{2}\right)\theta^{\beta k/2+\beta l}}\frac{t^{k/2(\beta-\alpha-1)+m+\alpha n+\beta l}}{\Gamma(k/2(\beta-\alpha-1)+r+\alpha n+\beta l+1)}+A\sum_{k=1}^{\infty}\frac{(-y)^k M^{k/2-1}}{v^{k/2}k!}$$

$$\sum_{r=0}^{\infty}\frac{M^{r+k/2-1}(-1)^r\Gamma\left(r-1+\frac{k}{2}\right)}{r!\Gamma\left(\frac{k}{2}-1\right)}\sum_{n=0}^{\infty}\frac{\lambda^{\alpha k/2-\alpha n}(-1)^n\Gamma\left(n-\frac{k}{2}\right)}{n!\Gamma\left(-\frac{k}{2}\right)}$$

$$\sum_{l=0}^{\infty}\frac{(-1)^l\Gamma\left(l+\frac{k}{2}\right)}{l!\Gamma\left(\frac{k}{2}\right)\theta^{\beta k/2+\beta l}}\frac{t^{k/2(\beta-\alpha-1)+r+\alpha n+\beta l+1}}{\Gamma(k/2(\beta-\alpha-1)+r+\alpha n+\beta l+2)} \tag{24}$$

Equation (24) can be written in simpler form ion terms of H-Fox function and as follows:

$$u = U + \sum_{k=1}^{\infty}\frac{(-y)^k M^{k/2}}{v^{k/2}k!}\sum_{m=0}^{\infty}\frac{M^{-k/2+m}(-1)^m}{m!}\sum_{n=0}^{\infty}\frac{\lambda^{\alpha k/2-\alpha n}(-1)^n}{n!}t^{k/2(\beta-\alpha-1)+m+\alpha n}$$

$$H_{3,5}^{1,3}\left[\frac{t^{\beta}}{\theta^{\beta}}\middle|\begin{matrix}\left(1-\frac{k}{2},1\right),\left(1-n+\frac{k}{2},0\right),\left(1-m+\frac{k}{2}\right)\\(0,1),\left(1+\frac{k}{2},0\right),\left(1+\frac{k}{2},0\right),\left(1-\frac{k}{2},0\right),(k/2(\alpha+1-\beta)-r-\alpha n,\beta)\end{matrix}\right]+$$

$$A\sum_{k=1}^{\infty}\frac{(-y)^k M^{k/2-1}}{v^{k/2}k!}\sum_{r=0}^{\infty}\frac{M^{r+k/2-1}(-1)^r}{r!}\sum_{n=0}^{\infty}\frac{\lambda^{\alpha k/2-\alpha n}(-1)^n}{n!}t^{k/2(\beta-\alpha-1)+r+\alpha n+1}$$

$$H_{3,5}^{1,3}\left[\frac{t^{\beta}}{\theta^{\beta}}\middle|\begin{matrix}\left(1-\frac{k}{2},1\right),\left(1-n+\frac{k}{2},0\right),\left(2-r-\frac{k}{2}\right)\\(0,1),\left(1-\frac{k}{2},0\right),\left(1+\frac{k}{2},0\right),\left(2-\frac{k}{2},0\right),(k/2(\alpha+1-\beta)-r-\alpha n+1,\beta)\end{matrix}\right] \tag{25}$$

where the H-Fox function [8] is defined as follows:

$$H_{p,q+1}^{1,p}\left[-x\middle|\begin{matrix}(1-a_1,A_1),\ldots,(1-a_p,A_p)\\(0,1),(1-b_1,B_1),\ldots,(1-b_q,B_q)\end{matrix}\right]=\sum_{k=0}^{\infty}\frac{\Gamma(a_1+A_1k)\ldots\Gamma(a_p+A_pk)}{k!\Gamma(b_1+B_1k)\ldots\Gamma(b_q+B_qk)}$$

By similar procedure the shear stress equation (12) can be solved. It is found the shear stress is of the form:

$$S_{xy} = -U\rho \sum_{k=1}^{\infty}\frac{(-y)^k M^{(k+1)/2}}{v^{(k-1)/2}k!}\sum_{m=0}^{\infty}\frac{M^{-((k+1)/2)+m}(-1)^m}{m!}$$

$$\sum_{n=0}^{\infty}\frac{\lambda^{\alpha((k-1)/2)-\alpha n}(-1)^n}{n!}t^{k/2(\beta-\alpha-1)+m+\alpha n-1}$$

$$H_{3,5}^{1,3}\left[\frac{t^{\beta}}{\theta^{\beta}}\left|\begin{matrix}\left(1-\frac{k-1}{2},1\right),\left(1-n+\frac{k-1}{2},0\right),\left(1-m-\frac{k+1}{2}\right)\\(0,1),\left(1-\frac{k-1}{2},0\right),\left(1+\frac{k-1}{2},0\right),\left(1+\frac{k+1}{2},0\right),(k/2(\alpha+1-\beta)-m-\alpha n+1,\beta)\end{matrix}\right.\right]+$$

$$A\rho\sum_{k=1}^{\infty}\frac{(-y)^k M^{(k-1)/2}}{v^{(k-1)/2}k!}\sum_{m=0}^{\infty}\frac{M^{-((k-1)/2)+m}(-1)^m}{m!}\sum_{n=0}^{\infty}\frac{\lambda^{\alpha((k-1)/2)-\alpha n}(-1)^n}{n!}t^{k/2(\beta-\alpha-1)+m+\alpha n}$$

$$H_{3,5}^{1,3}\left[\frac{t^{\beta}}{\theta^{\beta}}\left|\begin{matrix}\left(1-\frac{k-1}{2},1\right),\left(1-n+\frac{k-1}{2},0\right),\left(1-m+\frac{k-1}{2}\right)\\(0,1),\left(1-\frac{k-1}{2},0\right),\left(1+\frac{k-1}{2},0\right),\left(1+\frac{k-1}{2},0\right),(k/2(\alpha+1-\beta)-m-\alpha n,\beta)\end{matrix}\right.\right]$$

Results and Discussion

This section displays the graphical illustration velocity field for the flows analyzed in this investigation. We interpret these results with respect to the variation of emerging parameters of interest. The exact analytical solutions for accelerated flows have been obtained for the fractional Oldroyd-B fluid. The following results are made:

- As A increases there is decreasing in velocity, seen in figurer-1.
- As λ increases there is decreasing in velocity, seen in figure-2.
- As M increases there is decreasing in velocity, seen in figure-3.
- As θ increases there is increasing in velocity. seen in figure-4.
- As α increases there is decreasing in velocity, seen in figure-5.
- As β increases there is increasing in velocity, seen in figure-6.
- As t increases there is decreasing in velocity, seen in figure-7.
- As U increases there is increasing in velocity, seen in figure-8.

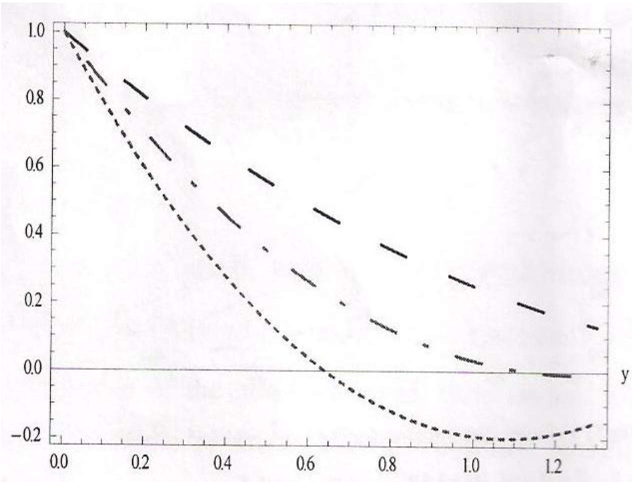

Figure.1- Velocity distribution for λ=8, M=5, θ=10, α=0.2, β=0.9, V=1, U=1, A (---,-.-.,----)=0,0.1,1.2.

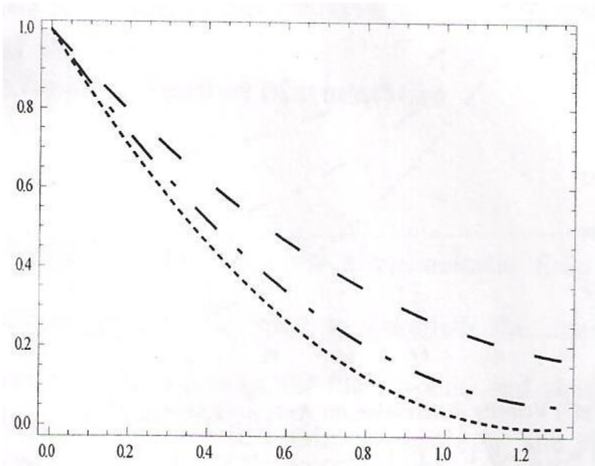

Figure.2- Velocity distribution for A=1, M=5, θ=10, α=0.2, β=0.9, V=1, U=1, λ (---,-.-.,----)=1,4,8.

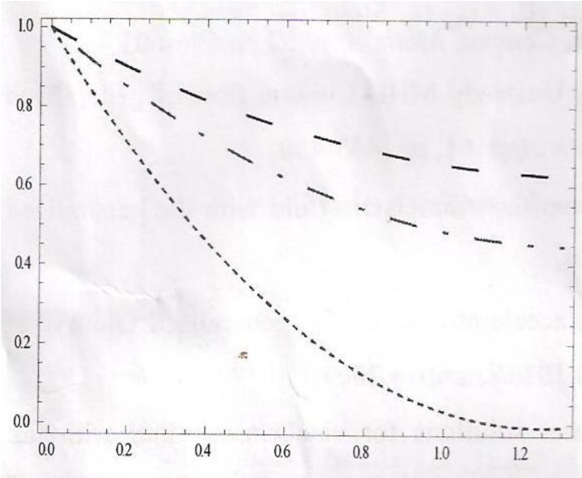

Figure.3- Velocity distribution for A=1, λ=8, θ=10, α=0.2, β=0.9, t=1, V=1, U=1, M (---,-.-.,----)=4,4.5,5.5.

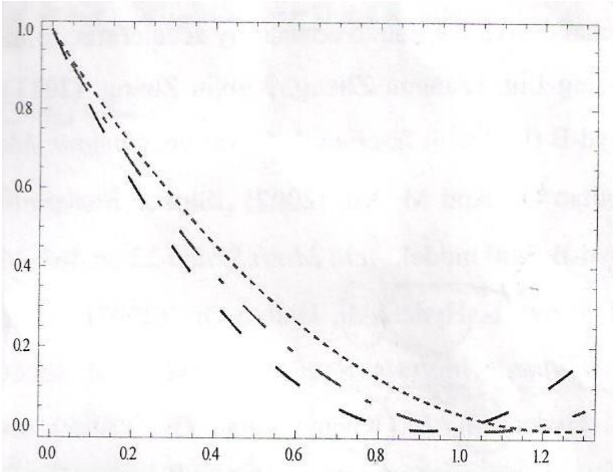

Figure.4- Velocity distribution for A=1, λ=8, M=5, α=0.2, β=0.9, t=1, V=1, U=1, θ (---,-.-.,----)=5,7,10.

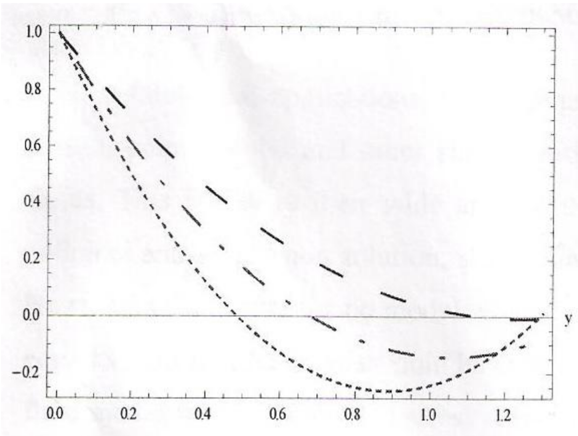

Figure.5- Velocity distribution for A=1, λ=8, M=5.5, θ=10,β=0.9, t=1, V=1, U=1, α (---,-.-.,----)=0.2,0.3,0.4.

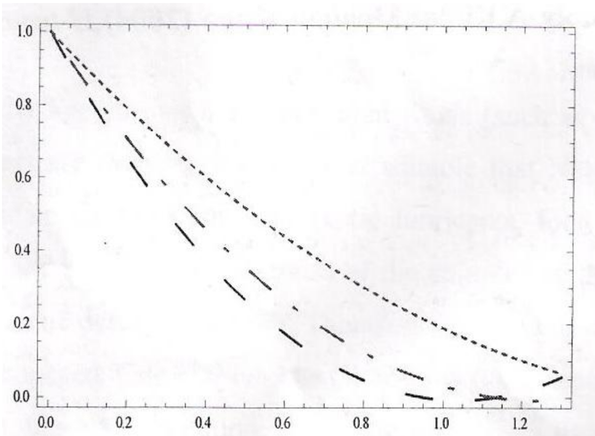

Figure.6- Velocity distribution for A=1, λ=8, M=5.5, θ=10,α=0.2, t=1, V=1, U=1, β (---,-.-.,----)=0.8,0.9,1.1.

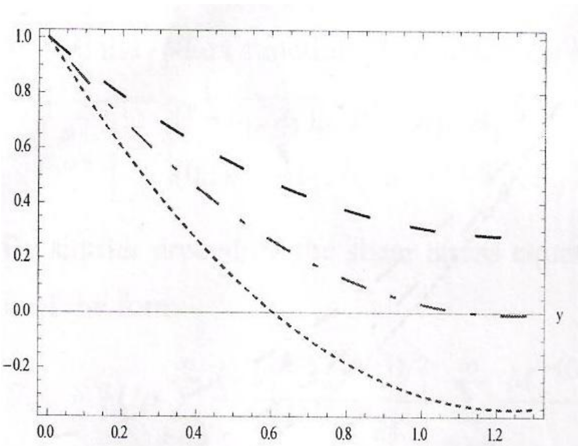

Figure.7- Velocity distribution for A=1, λ=8, M=5.5, θ=10,α=0.2, β=0.9, V=1, U=1, t(---,-.-.,----)=0.9,1,1.1.

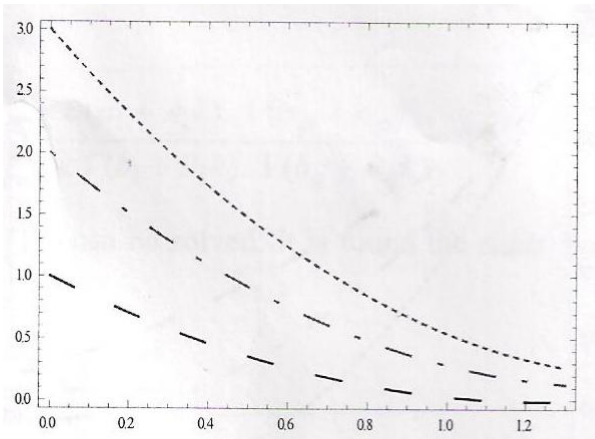

Figure.8- Velocity distribution for A=1, λ=8, M=5.5, θ=10,α=0.2, β=0.9, t=1,V=1, U (---,-.-.,----)=1,2,3.

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