Quasi Duo Rings whose Every Simple Singular Modules is YJ-Injective

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Abstract
In this paper , we give some characterizations and properties of Quasi duo rings whose every simple singular module is YJ-injective . and we study the relation between this rings and other rings , like NI-ring, non singular rings, generalized π-regular ring, strongly regular and n-regular ring .

Keyword: Quasi duo ring, YJ-injective rings, singular rings, non singular rings .

حلقات كوازي ديو والتي كل مقاس بسيط منفرد عليها غامر من النمط-YJ

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الخلاصة
في هذا البحث اعطينا بعض الصفات والخواص للحلقات من النمط quasi duo والتي كل مقاس منفرد بسيط فيها هو من النمط YJ ،وكذلك درسنا العلاقة بين هذه الحلقة والحلقات الاخرى مثل الحلقات من النمط NI،الحلقات غير المنفردة والحلقات المنتظمة بقوة والحلقات المنتظمة من النمط-n

Introduction

Throughout this paper R is associative ring with identity and all modules are unitary. For a subset X of R, the right (left) annihilator of X in R is denoted by $r(X)(l(X)).If\ X = \{a\}$, we usually abbreviated it to $r(a)(l(a))$. We write $J(R)$, for the Jacobson radical,

We call to the ring R is reduced if R is not contain any nilpotent element [1]. A ring R is said to be semiprime if R is not contain any nilpotent ideal [1].A ring R is said to be right weakly regular ring for each $a \in R$, there exists $b, c \in R$ such that $a = abac$ [2]. A ring R is said to be MERT if and only if every maximal essential right ideal of R is an two sided ideal [3].N(R) denoted the set of all nilpotent elements,$N_2(R)$ is the set of all elements such that $a^2 = 0$. A ring R is called NI, if N(R) is an ideal of R, A ring R is called strongly regular(π-regular, unit regular) if for every$a \in R$there exists $b \in R$, such that $a = a^2b(a^n = a^nba^n, a\ is\ unit\ element)$[4], A ring R is called n-regular (n-weakly regular) if $a \in aRa$, ($a \in RaRa$) for all $a \in N(R)$.

A right R-module M is called YJ-injective if for any $0 \neq a \in R$, there exists a positive integer n such that $a^n \neq 0$ and every right R-homomorphism of a^nR into M extends to one of R into M [5].YJ-injectivity is also called GP-injectivity, by several authors [6]. We call the ring R is quasi duo ring, if every maximal right ideal is a two sided ideal [7].

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Properties of quasi duo ring whose every simple singular module is YJ-injective.

In this section we give properties of quasi duo ring whose every simple singular right R-module is YJ-injective and its relation with the other ring.

Theorem 2.1

Let R be a right quasi duo ring whose every simple singular right R-module is YJ-injective. Then J(R) is nil ideal of R.

Proof:

Let $a \in J(R)$, for some positive integer n, either $a^nR + r(a^n)$ is essential or not, if $a^nR + r(a^n)$ is not essential in R, then there exists a right ideal K of R such that $a^nR + r(a^n) \oplus K$ is essential right ideal of R, if $a^nR + r(a^n) \oplus K \neq R$, then there exists a maximal right ideal M of R containing $a^nR + r(a^n) \oplus K$, since $a^nR + r(a^n) \oplus K$ is essential, so M is essential, we get that R/M is YJ-injective, then there exists a positive integer n and $a^n \neq 0$ such that any R-homomorphism of a^nR into R/M extends to one of R into R/M, let $f: a^nR \to R/M$ such that $f(a^nr) = r + M$, where $r \in R$, f is well define, since R/M is YJ-injective, there exists $c \in R$ such that $1 + M = f(a^n) = ca^n + M, 1 - ca^n \in M$, since R is a right quasi duo ring, so $ca^n \in M$ implies that $1 \in M$, which is a contradiction.

Therefore $a^nR + r(a^n) \oplus K = R$, then there exists $0 \neq e = e^2 \in R$, such that $a^nR + r(a^n) = eR$, $a^nb + v = e$, for some $b \in R$, and $v \in r(a^n), a^{2n}b = a^ne$, since $a^n \in eR$, implies that $a^n = ed$ for some $d \in R$, we get $a^{2n}bed = a^ned$, then $a^{2n}ba^n = a^{2n}, a^{2n}(1 - ba^n) = 0$, since $a^n \in J(R)$ implies that $1 - ba^n$ is invertible, if $1 - ba^n = 0$, we get that $1 \in J(R)$ which is contradiction, so must $a^{2n} = 0$, so a is nilpotent element. If $a^nR + r(a^n)$ is essential, then there exists a maximal right ideal X of R containing $a^nR + r(a^n)$, so X is essential, we have R/X is YJ-injective, similar to above, we have $a^nR + r(a^n) = R$, for some $r \in R$ and $z \in r(a^n)$, $a^nr + z = 1$, $a^n = a^{2n}r, a^n(1 - a^nr) = 0$, if $1 - a^nr = 0$, then $1 \in J(R)$, which is contradiction, then must $a^n = 0$, also a is nilpotent element. Therefore $J(R)$ is nil ideal.

Lemma 2.2[8]

If R is a right or left quasi duo ring. Then $N(R) \subseteq J(R)$.

Corollary 2.3

Let R be a right quasi duo ring. Then R/J(R) is reduced ring.

Proof:

From Lemma 2.2, $N(R) \subseteq J(R)$. Therefore R/J(R) is reduced ring.

Theorem 2.4

Let R be a right quasi duo ring whose every simple singular right R-module is YJ-injective. Then R is NI ring.

Proof:

To prove that R is NI ring, we to show that $N(R)$ is an ideal, so we to prove that $N(R) = J(R)$, since R is right quasi duo ring and by Lemma 2.2, we get $N(R) \subseteq J(R)$, from Theorem 2.1, we have $J(R) \subseteq N(R)$, therefore $N(R) = J(R)$, then R is NI ring.

Preposition 2.5

Let R be a right quasi duo ring whose every simple singular right R-module is YJ-injective. Then $r(a)$ is not essential right ideal for every $a \in N_2(R)$.

Proof:

Let $0 \neq a \in N_2(R)$, then $r(a) \neq 0$, if $r(a) = R$, then $a = 0$ which is contradiction with $a \neq 0$, then $r(a) \neq R$, so there exists a maximal right M of R containing $r(a)$, if $r(a)$ is essential right ideal of R, so M is essential, we get that R/M is YJ-injective, then there exists a positive integer n and $a^n \neq 0$, since $a^2 = 0$, so n=1, such that any R-homomorphism of aR into R/M extends to one of R into R/M, let $f: aR \to R/M$ such that $f(ar) = r + M$, where $r \in R$, f is well define, since R/M is YJ-injective, there exists $c \in R$ such that $1 + M = f(a) = ca + M, 1 - ca \in M$, since R is a right quasi duo ring, $ca \in M$, implies that $1 \in M$, which is a contradiction, then $r(a) = R$, implies that $a = 0$ which is also contradiction. Therefore $r(a)$ is not essential right ideal of R for every $a \in N_2(R)$.

Preposition 2.6

Let R be a right quasi duo ring whose every simple singular right R-module is YJ-injective. Then $r(a)$ is a direct summand of R for every $a \in N_2(R)$.

Proof:

Let $0 \neq a \in N_2(R)$, then $r(a) \neq 0$, by Preposition 2.5, $r(a)$ is not essential right ideal of R, then there exists a right ideal K of R, such that $r(a) \oplus K$ is an essential right ideal of R, if $r(a) \oplus K \neq R$, then there exists a maximal right ideal M of R containing $r(a) \oplus K$, since $r(a) \oplus K$ is an essential, so M is essential, that is mean M is a YJ-injective, as we shown in Preposition 2.5, we get a contradiction, therefore $r(a) \oplus K = R$, so $r(a)$ is a direct summand of R for every $a \in N_2(R)$

Lemma 2.7 :[9]

Let I be a right (left) ideal of a ring R, then R / I is a flat right (left) R-module if and only if for each a $\in$ I, there exists b $\in$ I such that a = ba (a= ab).

Corollary 2.8

Let R be a right quasi duo ring whose every simple singular right R-module is YJ-injective. Then $R/r(a)$ is flat right R-module for every $a \in N_2(R)$.

Proof:

Let $a \in N_2(R)$**,** then $r(a) \neq 0$, by Preposition 2.6, there exists a right ideal K of R such that $r(a) \oplus K = R$, then there exists $0 \neq e = e^2 \in R$, such that $r(a) = eR$, so $d = ed$ for all $d \in r(a)$, by Lemma(2.7), $R/r(a)$ is flat right R-module.

Y(R) is denoted to right singular ideal of R

Lemma 2.9 [10]

If $0 \neq$ Y(R), then there exists $0 \neq$ y $\in$ Y(R), such that $y^2 = 0$.

Theorem 2.10

Let R be a right quasi duo ring whose every simple singular right R-module is YJ-injective. Then R is right nonsingular ring.

Proof:

Let $0 \neq Y(R)$, by Lemma 2.9, we get that there exists $0 \neq a \in$ Y(R), such that $a^2 = 0$, so $r(a) \neq 0$, since $a \in$ Y(R), $r(a)$ is essential right ideal of R, since $a^2 = 0$, so $a \in N_2(R)$, by Preposition 2.6, we have that $r(a)$ is not essential which is a contradiction with $0 \neq Y(R)$. Therefore R is right nonsingular.

Theorem 2.11

Let R be a right quasi duo ring whose every simple singular right R-module is YJ-injective. Then R is generalized $\pi - regular$ ring.

Proof:

Let $a \in R$, for some positive integer n, if $a^nR + r(a^n) = R$, so there exists $r \in R$ and $v \in r(a^n)$ such that $a^nr + v = 1$, $a^nra^n + va^n = a^n$, $a^{2n} = a^{2n}ra^n$, set $d = ra^{n-1}$, $a^{2n} = a^{2n}da$. Therefore a is generalized $\pi - regular$ element. If $a^nR + r(a^n) \neq R$, then there exists a maximal right ideal M of R containing $a^nR + r(a^n)$, if $a^nR + r(a^n)$ is essential of R, so M is essential, we get that R/M is YJ-injective, then there exists a positive integer n and $a^n \neq 0$,such that any R-homomorphism of a^nR into R/M extends to one of R into R/M, let$f: a^nR \to R/M$ such that $f(a^nr) = r + M$, where $r \in R$, f is well define, since R/M is YJ-injective, there exists $c \in R$ such that $1 + M = f(a^n) = ca^n + M, 1 - ca^n \in M$, since R is a right quasi duo ring, $ca^n \in M$, implies that $1 \in M$, which is a contradiction, then $a^nR + r(a^n) = R$, similar to above we get that a is generalized $\pi - regular$ element. If $a^nR + r(a^n)$ is not essential right ideal of R, then there exists a right ideal K of R, such that $a^nR + r(a^n)$ is $\oplus K$ is an essential right ideal of R, if $a^nR + r(a^n) \oplus K \neq R$, then there exists a maximal right ideal L of R containing $a^nR + r(a^n) \oplus K$, since $a^nR + r(a^n) \oplus K$ is an essential, so L is essential, that is mean R/L is a YJ-injective, as we shown in above, we get that $a^nR + r(a^n) \oplus K = R$. Therefore $a^nR + r(a^n)$ is a direct summand right ideal generated by idempotent element, then there exists $0 \neq e = e^2 \in R$, $a^nR + r(a^n) = eR$, then $a^n = ed$ for some $d \in R$, $a^nb + v = e$, since $v \in r(a^n)$, $a^{2n}b + a^nv = a^ne$, a^ne^2d, $a^{2n}bed = a^ned$, $a^{2n}ba^n = a^{2n}$, set $w = ba^{n-1}$, $a^{2n} = a^{2n}wa$. Therefore a is generalized $\pi - regular$ element. So R is a generalized $\pi - regular$ ring.

Lemma 2.12 [11]

Let R be a ring. Then the following are equivalent.

1- R is regular ring.
2- R is generalized $\pi - regular$ ring and $N_2(R)$ is regular.

Theorem 2.13

R is strongly regular ring if and only; if R is quasi duo ring whose every simple singular right R-module is YJ-injective and $N_2(R)$ is regular .

Proof:

Let R is strongly regular ring, then proof is clearly.

Conversely, from Theorem 2.11, we get that R is generalized $\pi - regular$ ring, since $N_2(R)$ is regular, by Lemma 2.12, we have R is regular ring, since R is quasi duo ring and by Lemma 2.2, $N(R) \subseteq J(R) = 0$,(since R is regular ring $J(R) = 0$) implies that $N(R) = 0$, so R is reduced ring. Hence R is strongly regular ring.

Lemma 2.14 [12]

Let R be a right quasi duo ring, then the following are equivalent:

1- R is strongly regular.
2- R is right weakly regular.

Theorem 2.15

Let R be a right quasi duo ring whose every simple singular right R-module is YJ-injective. Then $R/J(R)$ is strongly regular ring.

Proof*:*

Let $J(R) = \bar{0} \neq \bar{a} \in \bar{R} = R/J(R)$, where $\bar{a} = a + J(R)$, if $\bar{R}\bar{a}\bar{R} + r(\bar{a}) \neq \bar{R}$. Suppose that it is not, then there exists a maximal right ideal M of R such that $\bar{R}\bar{a}\bar{R} + r(\bar{a}) \subseteq M/J(R)$, if $\bar{R}\bar{a}\bar{R} + r(\bar{a})$ is not essential in $\bar{R}$, then there exists a right ideal $\bar{I} = I/J(R)$ such that $\bar{R}\bar{a}\bar{R} + r(\bar{a}) \cap \bar{I} = \bar{0}$, then $\bar{I}\bar{a} \subseteq \bar{R}\bar{a}\bar{R} \cap \bar{I} = \bar{0}$, so $\bar{I} \subseteq l(\bar{a}) \subseteq r(\bar{a})$ ($\bar{R}$ is reduced ring, from corollary 2.3, since $N(R) = J(R)$, $R/N(R) = R/J(R)$, so $\bar{R}$ is reduced ring). Hence $\bar{I} = \bar{0}$, whence $\bar{R}\bar{a}\bar{R} + r(\bar{a})$ is an essential right ideal of $\bar{R}$. then must M is essential right ideal of R. Therefore R/M is a simple singular right R-module, so R/M is YJ-injective, then there exists a positive integer n and $a^n \neq 0$,such that any R-homomorphism of a^nR into R/M extends to one of R into R/M, let$f: a^nR \to R/M$ such that $f(a^n r) = r + M$, where $r \in R$, f is well define, since $\bar{R}$ is reduced. R/M is YJ-injective, there exists $c \in R$ such that $1 + M = f(a^n) = ca^n + M, 1 - ca^n \in M$, since R is a right quasi duo ring, $ca^n \in M$, implies that $1 \in M$, which is a contradiction. Hence $\bar{R}\bar{a}\bar{R} + r(\bar{a}) = \bar{R}$, and that is for all $\bar{a} \in \bar{R}$. Therefore $\bar{R}$ is right weakly regular ring. Since $\bar{R}$ is quasi duo ring and right weakly regular ring then by Lemma 2.14, we get $\bar{R} = R/J(R)$ is a strongly regular ring.

Lemma 2.16 :[13]

Let R be n-regular ring then $N(R) \cap J(R) = 0$.

Corollary 2.17

if R is quasi duo ring and n-regular whose every simple singular right R-module is YJ-injective , then R is strongly regular ring

Proof:

Since R is n-regular ring then by Lemma 2.16, $N(R) \cap J(R) = 0$, but by Theorem 2.1 J(R)$\subseteq N(R) \cap J(R) = 0$, which implies $J(R) = 0$, since $R/J(R)$ is strongly regular ring by Theorem 2.14, and $R/J(R) \cong R/\{0\} = R$, therefore R is strongly regular ring.

Another proof, since R is n-regular ring then $N_2(R)$ is regular, by Theorem 2.13, we get that R is strongly regular ring.

Lemma 2.18 :[13]

Let R be n-weakly regular ring then $N(R) \cap J(R) = 0$.

Corollary 2.19

R is strongly regular ring if and only if R is quasi duo ring and n-weakly regular whose every simple singular right R-module is YJ-injective.

Proof:

Similar to corollary 2.17 and by using Lemma 2.18.

Recall that a ring R is called strongly $\pi - regular$ if for every a in R there exists a positive integer n, depending on a, and an element x in R satisfying $a^n = a^{n+1}x$ [14].

Theorem 2.20

Let R be a right quasi duo ring whose every simple singular right R-module is YJ-injective. Then R is strongly $\pi - regular$ ring if R is bounded index of nilpotency.

Proof:

Let n be the bounded index of nilpotency of ring, from Theorem 2.15, we have $R/J(R)$ is strongly regular, then $\bar{a}\bar{R} = \overline{a^2}\bar{R} = \overline{a^3}\bar{R} \dots \overline{a^{n+1}}\bar{R}$, then $a - a^{n+1}b \in J(R)$ for some $b \in R$. Since $J(R)$ is nil by Theorem 2.1, it follows that $(a - a^{n+1}b)^n = 0$, we have

$a^n = a^{n-1}(a^{n+1}b) - a^{n-2}(a^{n+1}b)^2 \dots (a^{n+1}b)^n = a^{n+1}[a^{n-1}b - a^{n-2}b(a^{n+1}b) \dots b(a^{n+1}b)^{n-1}]$

Set $d = a^{n-1}b - a^{n-2}b(a^{n+1}b) \dots b(a^{n+1}b)^{n-1}$, then $a^n = a^{n+1}d$ for all $a \in R \setminus J(R)$, when $a \in J(R)$, so clearly that $a^n = 0 = a^{n+1}r$ for any $r \in R$. Therefore R is strongly $\pi - regular$ ring.

Corollary 2.21

Let R be a right quasi duo ring whose every simple singular right R-module is YJ-injective. Then R is strongly $\pi - regular$ ring if $J(R)$ is nilpotent ideal.

Proof:

From Theorem 2.20

A ring R is called an (S,2)-ring if every element in R is a sum of two units in R [15].

Lemma 2.22, [4]

Let R be a strongly regular ring then R is unit regular.

Theorem 2.23

Let R be a right quasi duo ring whose every simple singular right R-module is YJ-injective. R is an (S,2)-ring if and only if every idempotent element in R is a sum of two units in R.

Proof:

Let R be an (S,2)-ring , then it is clearly that every idempotent element in R is a sum of two units in R.

Converse

Let $a \in R$, by Theorem 2.15, we get that R/J(R) is strongly regular ring by Lemma 2.22,

Then there exists a unit $u + J(R) \in R/J(R)$ such that

$a + J(R) = [a + J(R)][u + J(R)][a + J(R)] \quad = aua + J(R)$,

now $[au + J(R)]^2 = auau + J(R)$

$\qquad = au + J(R)$,

so $au + J(R)$ is an idempotent element in $R/J(R)$ by [16](12, proposition 1,p. 72) there exists $e \in R$ such that $au + J(R) = e + J(R)$. Then

$a + J(R) = auu^{-1} + J(R)$,

$=[au + J(R)][u^{-1} + J(R)]$

$=[e + J(R)][u^{-1} + J(R)]$

$=eu^{-1} + J(R)$

Therefore $a + j_1 = eu^{-1} + j_2$ where $j_1, j_2 \in J(R)$

$a = eu^{-1} + j_2 - j_1$, set $j = j_2 - j_1$

$a = eu^{-1} + j$, for some $j \in J(R)$

where $u^{-1} \in R$ is the multiplicative inverse of u by hypothesis e=v+w where e is idempotent and v,w is units in R, so $a = (v + w)u^{-1} + j = vu^{-1} + wu^{-1}j$, vu^{-1} is unit sine $vu^{-1}uv^{-1} = 1$, and also $uv^{-1}vu^{-1} = 1$, $wu^{-1} + j$ is unit

since $(wu^{-1} + j)uw^{-1}(1 + juw^{-1})^{-1} = (wu^{-1}uw^{-1} + juw^{-1})(1 + juw^{-1})^{-1} = (1 + juw^{-1})(1 + juw^{-1})^{-1} = 1$,

it is clear that u, v, w is unit, but $1 + juw^{-1}$ is invertible because $juw^{-1} \in J(R)$, so $1 + juw^{-1}$ is invertible

$(1 + juw^{-1})^{-1}uw^{-1}(wu^{-1} + j) = (1 + juw^{-1})^{-1}(uw^{-1}wu^{-1} + uw^{-1}j \quad = \quad (1 + juw^{-1})^{-1}(1 + uw^{-1}j)$ =1

So vu^{-1} and $(wu^{-1} + j)$ is a unit, so a is a sum of two units in R. Therefore R is an (S,2) –ring .

A ring R is called P.I. ring if R satisfies a polynomial identity with coefficients in the ring of integers and at least one of them either 1 or -1 [17].

Lemma 2.24 [17]

For a P.I. ring R the following condition are equivalent:

1- R is strongly $\pi - regular$.
2- R is $\pi - regular$
3- Every prime ideal of R is maximal.

4- Every prime factor ring of R is von Neumann regular.

Theorem 2.25

Let R be a right quasi duo ring whose every simple singular right R-module is YJ-injective. Then R is strongly $\pi - regular$ *if R is P.I. ring.*

Proof:

Let R be not strongly $\pi - regular$ ring. Then there is a prime ideal P of R such that , the prime factor ring R/P is not regular ring by Lemma 2.24, by Theorem 2.15, R/J(R) is strongly regular ring, hence it is regular ring. If $J(R) \subseteq P(R)$, we define the mapping $f: R/J(R) \longrightarrow R/P$ by $f\big(a + J(R)\big) = a + P$, it is clearly that f is homomorphism and the mapping is onto, so f is homomorphic, we get that R/P is regular which is a contradiction with R/P is not regular, therefore $J(R) \nsubseteq P(R)$, then there exist an $a \in J(R)$ and $a \notin P(R)$, $a + P \neq P$, since $a \in J(R)$ and $J(R)$ is nil by Theorem 2.1, then there exist a positive integer n such that $a^n = 0$, so $(a + P)^n = a^n + P = P$, which implies $a^n \in P$, hence P is a prime $a \in P$, which is also contradiction. Therefore R/P is regular, hence by Lemma 2.17, R is strongly $\pi - regular$ ring.

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