Influence of Thermal Radiation and MHD on the Boundary Layer Flow Due to an Exponentially Stretching Sheet

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Abstract
In this paper, the effect of thermal radiation and magnetic field on the boundary layer flow and heat transfer of a viscous fluid due to an exponentially stretching sheet is proposed. The governing boundary layer equations are reduced to a system of ordinary differential equations. The homotopy analysis method (HAM) is employed to solve the velocity and temperature equations.

Keywords: Steady flow, Exponentially stretching surface, Magnetohydro dynamics(MHD), HAM solution.

تأثير الاشعاع الحراري والمجال المغناطيسي على جريان الطبقة الحدودية لصفيحة متمددة اسياً

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الخلاصة:
في هذا البحث ، سنقوم بدراسة تأثير الاشعاع الحراري و المجال المغناطيسي على جريان ا لطبقة الحدودية والانتقال الحراري لمائع لزج على صفيحة متمددة اسيا ″. معادلات الطبقة الحدودية المنتظمة خفضت الى منظومة من ا لمعادلات ا لتفاضلية الاعتيادية . تم استخدام طريقة (HAM) لحل معادلتي السرعة والحرارة.

1. Introduction

The problem of incompressible flow of viscous fluid and heat transfer due to stretching surface has interesting numerous industrial applications such as wired drawing,oerodynamic extrusion of plastic sheets, the cooling presses of metallic plate in a cooling bath and glass and polymer industries ,the boundary layer along a liquid film. Sakiad [1] was the first to study the boundary layer flow on continuous solid surface. He derived the basic differential integral momentum equations. Elbashbeshy[2] study the similarity solutions of the laminar boundary layer equations describing heat and flow by an exponentially stretching surface subject to suction. The thermal boundary layer on an exponentially stretching continuous surface with an exponential temperature distribution, in the presence of the magnetic field effect is investigated numerically by Al-odat et al.[3].The effect of radiation on the boundary layer flow and heat transfer of a viscous fluid over an exponentially stretching sheet is studied by Sajid and Hayat [4]. Elbashbeshy , Emamb and Abdelgaberc .[5] discussed the effects of thermal radiation and magnetic field on unsteady flow and heat transfer over an exponentially stretching surface in the presence of internal heat generation/absorption.

In this paper, we discuss the effect of thermal radiation and magnetic field on the steady , two-dimensional boundary layer flow and heat transfer over an exponentially stretching sheet .Homotopy analysis method(HAM)[6-8] is used in obtaining the analytic solution.

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2. The homotopy analysis method (HAM)

We consider the following differential equation

$N[u(\tau)] = 0$ ..(1)

where N is a nonlinear operator, τ denotes independent variables, u (τ) is an unknown function, respectively. For simplicity, we ignore all boundary or initial conditions, which can be treated in the similar way. By means of generalizing the traditional homotopy method, constructs the so called zero – order deformation equation

$(1-p)L[\Phi(\tau;p)-u_0(\tau)]=phH(\tau)N[\Phi(\tau;p)]$ ..(2)

where $p\in[0,1]$ is the embedding parameter, $h\neq 0$ is a nonzero parameter, H($\tau\neq 0$) is an auxiliary function, L is an auxiliary linear operator, $u_0(\tau)$ is an initial guess , $\Phi(\tau,p)$ is a unknown function respectively. It is important, that one has great freedom to choose auxiliary things in HAM. Obviously, when p=0 andp=1, it holds

$u_0(\tau)=\Phi(\tau;0),\ u_1(\tau)=\Phi(\tau;1)$ ..(3)

Thus, as p increases from 0 to 1, the solution $\Phi(\tau;p)$ varies from the initial guesses $u_0(\tau)$ to the solution u (τ) . Expanding $\Phi(\tau;p)$ in Taylor series with respect to p, we have

$$\Phi(\tau;p) = u_0(\tau)+\sum_{m=1}^{\infty} u_m(\tau)\ p^m \quad ..(4)$$

$$u_m = \frac{1}{m!}\frac{\partial^m\Phi(t,p)}{\partial p^m}\Big|_{p=0} \quad (5)$$

If the auxiliary linear operator, the initial guess, the auxiliary h, and the auxiliary function are so properly chosen, the series (4) converges at p=1 , then we have

$$u(\tau) = u_0(\tau)+\sum_{m=1}^{\infty} u_m(\tau) \quad ..(6)$$

Let us define the vector

$\vec{u}(\tau)=\{u_0(\tau), u_1(\tau), \ldots\ldots u_n(\tau)\}$

Differentiating equation (2) m times with respect to the embedding parameter p and then setting p = 0 and finally dividing them by m! we obtain the mth – order deformation equation

$$L[u_m(\tau) - x_m u_{m-1}] - hH(\tau)R_m(\overrightarrow{u_{m-1}}) = 0 \quad ..(7)$$

where

$$R_m(\overrightarrow{u_{m-1}}) = \frac{\partial^{m-1}N[\Phi(\tau;p)]}{\partial p^{m-1}}\Big|_{p=0} \quad ..(8)$$

and

$$x_m=\begin{cases}0, m\leq 1\\ 1, m>1\end{cases}$$

Applying L^{-1} on both side of equation (7), we get

$u_m(\tau) = x_m u_{m-1}(\tau)+ hL^{-1}\ [H(\tau)R_m(\tau)]$ ..(9)

In this way, it is easily to obtain u_m for $m\geq 1$, at mth- order, we have

$$u(\tau) = \sum_{m=0}^{M} u_m(t) \quad ..(10)$$

When $M\to\infty$, we get an accurate approximation solution of the original equation (1).

3. Formulation of the problem

Consider the flow subject to magnetic field of an incompressible viscous fluid bounded by a stretching surface, the x-axis is taken along the stretching surface in the direction of the motion and y-axis is perpendicular to it. The flow and heat transfer are governed by the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad ..(11)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\beta_0^2}{\rho}u \quad ..(12)$$

$$\rho c_\rho\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2-\frac{\partial q_r}{\partial y} \quad ..(13)$$

where u and v are the velocitics in the x-and y directions, respectively, ρ is the fluid density, $\upsilon=\frac{\mu}{\rho}$ is the kinematic , μ is the dynamic, σ is the electric conductivity,$\beta_0$is the strength of magnetic field ,T is the temperature, k is the thermal conductivity,$c_\rho$is the specific heat and q_r is the radiative heat flux.

The boundary conditions are

$$u(0)=u_0 \ , T(0)=T_\infty+T_0e^{x/2l}, \ u\to 0 \ , T\to 0 \text{ as } y\to\infty \ , v(0)=0 \ , y\to\infty \quad ..(14)$$

where u_0 is the reference velocity,T_0,T_∞ are respectively the temperatures at end far away from the plate, l is a constant.

By Rosseland approximation [3] of radiation for optically thick layer one has

$$q_r=\frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \quad ..(15)$$

where k* is the mean absorption coefficient and $\sigma^*$is the steran-boltzmann constant,we can write T^4 as a linear function then

$$T^4=4T_\infty^3T-3T_\infty^4 \quad ..(16)$$

By using equations (13) ,(15) and (16) the heat transfer equation can be written as:

$$\rho c_\rho\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\left(k+\frac{16\sigma^*}{3k^*}T_\infty^3\right)\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \quad ..(17)$$

Introducing the new variables

$$u=u_0e^{\frac{x}{l}}f'(\eta),\ v=-\sqrt{\frac{\nu u_0}{2l}}\,e^{\frac{x}{l}}\{f(\eta)+\eta f'(\eta)\}$$

$$T=T_\infty+T_0e^{\frac{x}{2l}}\theta(\eta),\ \eta=\sqrt{\frac{u_0}{2\nu l}}\,e^{\frac{x}{2l}}y \quad (18)$$

It's clear that equation (11) is satisfied and by substituting (18) in equation (12), (17), we get

$$f'''-Mf'+ff'-2(f')^2=0 \quad ..(19)$$

$$\left(1+\left(\frac{4k}{3}\right)\right)\theta''+pr\,(f\theta'-\theta f'+E(f'')^2)=0 \quad ..(20)$$

where pr=$\mu c_\rho/k$is Prandtle number,E=$u_0^2/(T_0c_\rho)$is Eckart number ,k= $4\sigma^*T_\infty^3/(k^*k)$is Radiation Number, M=$2L\sigma\beta_0^2e^{\frac{-x}{L}}/(\rho u_0)$ is magnatic parameter and prime denotes differentiation with η.

4. Method of the solution

We apply HAM method to find the solution of equations (9), (10). Assume the initial guess

$f_0(\eta)=1-e^{-\eta}$,

$\theta_0(\eta)=e^{-\eta}$ and the auxiliary linear operator

$L_1(f)=f'''-f'$, $L_2(\theta)=\theta''-\theta$ which satisfy

$L_1(C_1+C_2e^{\eta}+C_3e^{-\eta})=0$

$L_2(C_4e^{\eta}+C_5e^{-\eta})=0$

where $C_1,\ldots,C_5$ are constants to be determined from the boundary conditions.

We define the non linear operator

$$N_1[f(\eta,p)]=\frac{\partial^3 f(\eta,p)}{\partial\eta^3}+2\left(\frac{\partial f(\eta,p)}{\partial\eta}\right)^2+f(\eta,p)\frac{\partial^2 f(\eta,p)}{\partial\eta^2}-M\frac{\partial f(\eta,p)}{\partial\eta} \quad ..(21)$$

$$N_2[\theta(\eta,p),f(\eta,p)]=(1+4k/3)\frac{\partial^2\theta(\eta,p)}{\partial\eta^2}+Pr\left[f(\eta,p)\frac{\partial^2\theta(\eta,p)}{\partial\eta^2}-\frac{\partial f(\eta,p)}{\partial\eta}\theta(\eta,p)+E\left(\frac{\partial^2 f(\eta,p)}{\partial\eta^2}\right)^2\right] \quad ..(22)$$

Where p$\in[0,1]$is an embedding parameter, $\theta(\eta,p)$, $f(\eta,p)$are real functions of η,p.

Let h_1,h_2 denote the non- zero auxiliary parameters, we can construct the zero order Deformation equations

$$(1-p)L_1[f(\eta,p)-f_0(\eta)]=ph_1N_1[f(\eta,p)] \quad ..(23)$$

$$(1-p)L_2[\theta(\eta,p)-\theta_0(\eta)]=ph_2N_2[\theta(\eta,p)], \quad ..(24)$$

$f(0,p)=0,\ f'(0,p)=1,\ f'(\infty,p)=0,\ \theta(\infty,p)=0,\ \theta(0,p)=1$

For p=0,p=1 we have respectively

$f(\eta,0)=f_0(\eta), f(\eta,1)=f(\eta), \theta(\eta,0)=\theta_0(\eta),\ \theta(\eta,1)=\theta(\eta)$ ..(25)

Thus, as p increase from 0 to 1 f,θ varies from $f_0(\eta), \theta_0(\eta)$ to $f(\eta),\theta(\eta)$.

By Taylors theorem and equation (25), we have

$$f(\eta,p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m\, p^m \quad ..(26)$$
$$\theta(\eta,p) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m\, p^m$$

where

$$f_m = 1/m! \frac{\partial^m f(\eta,p)}{\partial p^m},$$
$$\theta_m = 1/m! \frac{\partial^m \theta(\eta,p)}{\partial p^m}$$

Assume that h_1, h_2 are selected such that the series (26) is convergent at p=1, then, owing to (25), we get

$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta), \theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta)$ ..(27)

Now differenting m times the zeroth order deformation equations (23),(24) with respect to p and then dividing by m ! and finally setting p=0 we get the following mth order deformation problems

$L_1[f_m - x_m f_{m-1}] = h_1 R^1_m$ ..(28)

$L_2[\theta_m - x_m \theta_{m-1}] = h_2 R^2_m$

$f_m(0) = f'_m(0) = f'_m(\infty) = \theta_m(0) = \theta_m(\infty) = 0$ ..(29)

$$R_m = f'''_{m-1} - M f'_{m-1} + \sum_{i=0}^{m-1} (f_{m-i-1} f''_{m-i-1} - 2 f'_i f'_{m-i-1}) \quad ..(30)$$

$$R_m = \left(1 + \left(\frac{4k}{3}\right)\right) \theta''_{m-1} + pr \sum_{i=0}^{m-1} (f_{m-i-1}\theta'_i - \theta_i f'_{m-i-1} + E f''_{m-i-1} f''_i) \quad ..(31)$$

$$x_m = \begin{cases} 0, m \le 1 \\ 1, m > 1 \end{cases}$$

By using software Mathematica it is found that:

$f_1=1/6\, e^{-2\eta} h - 3/4 e^{-\eta} h M - 1/2\, e^{-\eta} h M \eta - e^{-\eta} (1/12 (4 h-3 h M)) + 1/6 (h+3 h M)$. ..(32)

$f_2=1/6\, e^{-2\eta} h - 1/48\, e^{-3\eta} h^2 + 2/9\, e^{-2\eta} h^2 - 1/8\, e^{-\eta} h^2 - 3/4\, e^{-\eta} h M + 1/4\, e^{-2\eta} h^2 M - e^{-\eta} h^2 M - 7/16\, e^{-\eta} h^2 M^2 - 1/12\, e^{-\eta} h^2 \eta - 1/2\, e^{-\eta} h M \eta + 1/6\, e^{-2\eta} h^2 M \eta - 2/3\, e^{-\eta} h^2 M \eta - 3/8\, e^{-\eta} h^2 M^2 \eta - 1/8\, e^{-\eta} h^2 M^2 \eta^2 - e^{-\eta} (1/144 (48 h+49 h^2-36 h M-9 h^2 M^2)) + 1/72 (12 h+19 h^2+36 h M+54 h^2 M+27 h^2 M^2)$ ...(33)

$f_3=1/6\, e\text{-}2\, \eta\, h - 1/24\, e^{-3\eta} h^2 + 4/9\, e^{-2\eta} h^2 - 1/4\, e^{-\eta} h^2 + (11\, e^{-4\eta} h^3)/4320 - 29/576\, e^{-3\eta} h^3 + 47/144\, e^{-2\eta} h^3 - 29/96\, e^{-\eta} h^3 - 3/4\, e^{-\eta} h M + 1/2\, e^{-2\eta} h^2 M - 2\, e^{-\eta} h^2 M - 5/96\, e^{-3\eta} h^3 M + 23/36\, e^{-2\eta} h^3 M - 299/192\, e^{-\eta} h^3 M - 7/8\, e^{-\eta} h^2 M^2 + 5/16\, e^{-2\eta} h^3 M^2 - 61/48\, e^{-\eta} h^3 M^2 - 11/32\, e^{-\eta} h^3 M^3 - 1/6\, e^{-\eta} h^2 \eta + 1/36\, e^{-2\eta} h^3 \eta - 29/144\, e^{-\eta} h^3 \eta - 1/2\, e^{-\eta} h M \eta + 1/3\, e^{-2\eta} h^2 M \eta - 4/3\, e^{-\eta} h^2 M \eta - 1/32\, e^{-3\eta} h^3 M \eta + 7/18\, e^{-2\eta} h^3 M \eta - \ldots$ ..(34)

$\theta_1 = -(1/4)\, e^{-\eta} h - 1/3\, e^{-\eta} h k + 1/3\, e^{1-2\eta} h\, pr + 1/4\, e^{-\eta} h\, pr - 1/2\, e^{-\eta} h \eta - 2/3\, e^{-\eta} h k \eta + 1/2\, e^{-\eta} h\, pr\, \eta + e^{-\eta} (1/12 (3 h+4 h k-3 h\, pr-4 e h\, pr))$. ..(35)

$\theta_2 = -(1/4)\, e^{-\eta} h - 3/16\, e^{-\eta} h^2 - 1/3\, e^{-\eta} h k - 1/2\, e^{-\eta} h^2 k - 1/3\, e^{-\eta} h^2 k^2 + 1/3\, e^{1-2\eta} h\, pr + 1/4\, e^{-\eta} h\, pr - 1/6\, e^{1-3\eta} h^2\, pr + \ldots$ ..(36)

$\theta_3 = -(1/4)\, e^{-\eta} h - 3/8\, e^{-\eta} h^2 - 5/32\, e^{-\eta} h^3 - 1/3\, e^{-\eta} h k - e^{-\eta} h^2 k - 5/8\, e^{-\eta} h^3 k - 2/3\, e^{-\eta} h^2 k^2 - 5/6\, e^{-\eta} h^3 k^2 - 10/27\, e^{-\eta} h^3 k^3 + \ldots$ ..(37)

5. Result and discussion

In this section we show the convergence of the HAM solutions by choosing the values of h_1 and h_2 for h curves which ensure the convergence of the solutions , the effect of magnetic field parameter M depend upon velocity f and the temperature θ ,also the effect of thermal radiation k, prandtle number pr on the temperature θ are discussed in figures-(1,2)the h-curves are shown for the range of admissible value of h_1,h_2 it is found that the solution given by (32 ,33 …….37) ,converge in whole region of η when $h_1=-0.6, h_2=-0.5$.

Figure-3, shows variation of the functions f' and f. Figure-4, illustrate the effect of magnetic field parameter M upon the velocity for M= [0, 1, 2], its note that as M increase the velocity decrease. Figure-5, illustrate the influence of thermal radiation k on the temperature θ for k =[0,0.5,1], M=1,Pr=1,it is noted that with increasing k the temperature θ is decrease when $0.5 \geq \eta \geq 0$ and its increase when $\eta > 0.5$.

Figure-6, illustrate the effect of prandtle number Pr on temperature θ for Pr=[1,2,3], k=1, M=1,it is seen from this figure that when $\eta > 1$ the temperturre θ is decrease and its increasing for $\eta < 1$. The effect of magnetic field parameter M on temperature θ is shown in figure-7, its noted that the increase in the M decreases the temperature. From all figs (4-7) it's noted that the boundary condition is satisfied.

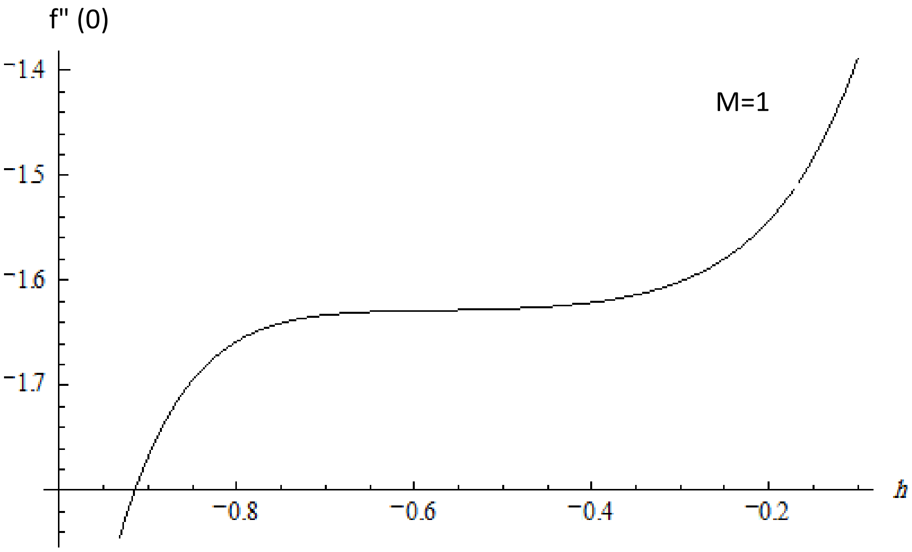

Figure 1- h-curve for function f

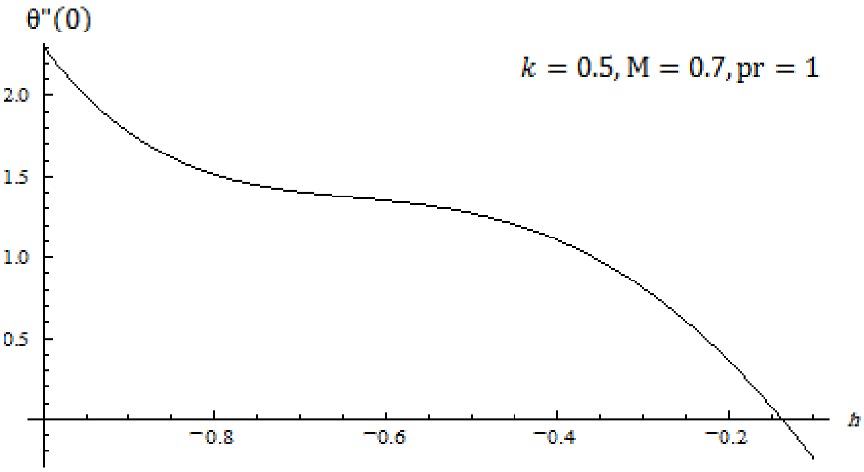

Figure 2- h-curve for function θ

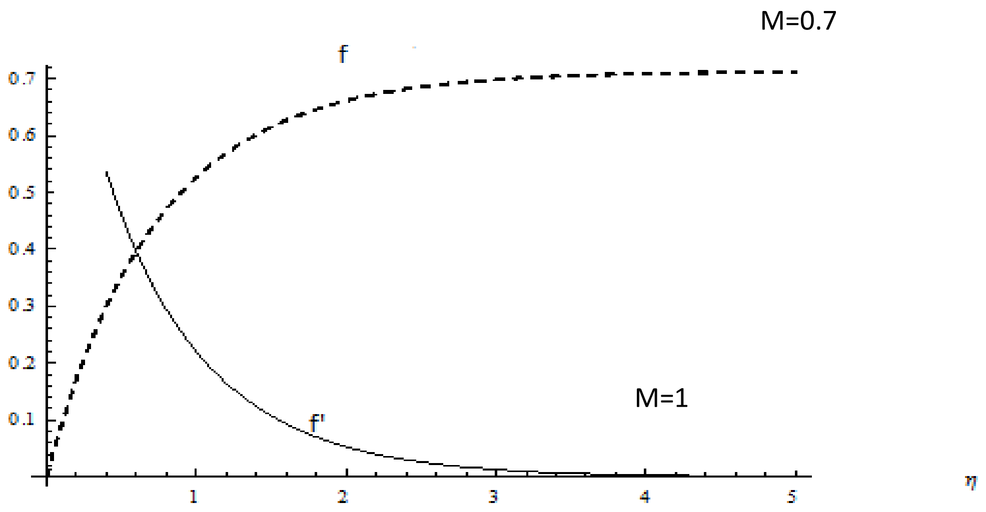

Figure 3- Velocity fields f' and f

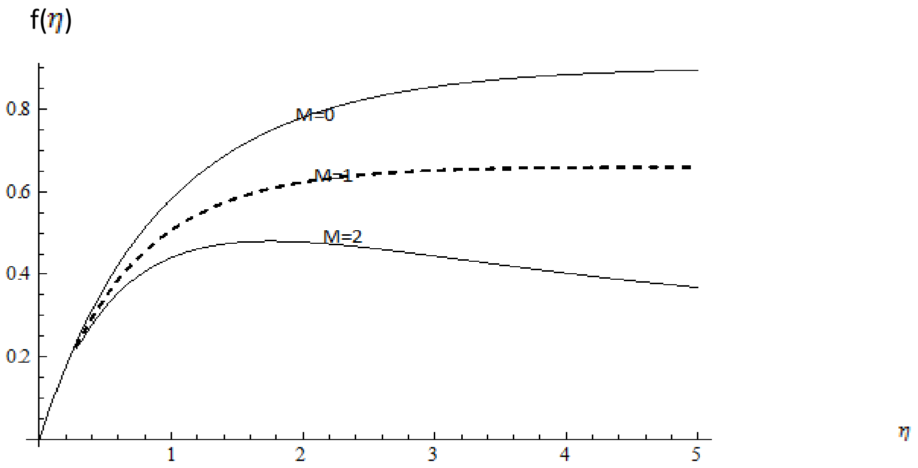

Figure 4- Influence of M on the velocity for M=[0, 1,2],h=-0.6

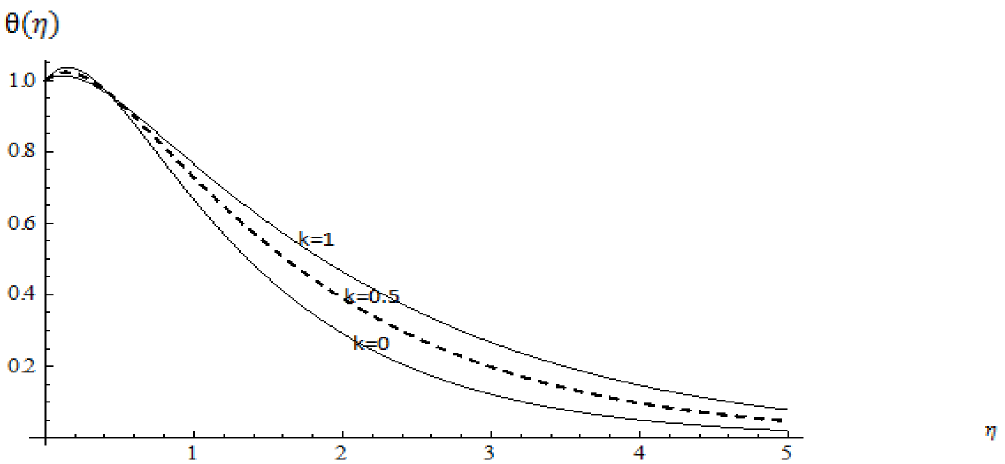

Figure 5- Influence of K on the temperature for K=[0,0.5,1],Pr=1,M=1,h=-0.5

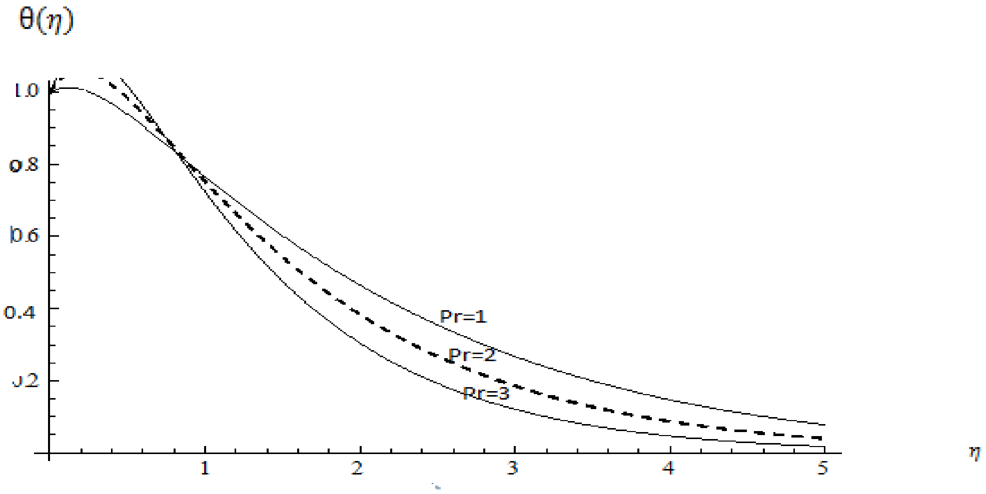

Figure 6- Influence of Pr on the temperature for Pr=[1,2,3],K=1,M=1,h=-0.5

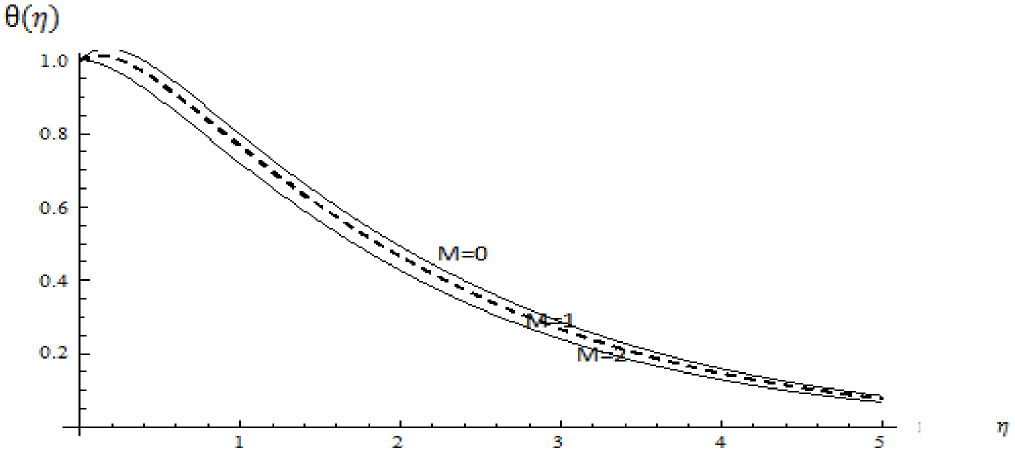

Figure 7- Influence of M on the temperature for M=[0,1 ,2],Pr=1,K=1,h=-0.5

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