



## Bayesian Analysis of five Exponentiated Distributions under different Priors and loss functions

Nada S. Karam\*

Department of Mathematics, College of Education, University of Al-Mustanseriya, Baghdad, Iraq.

### Abstract

The paper is concerned with posterior analysis of five exponentiated (Weibull, Exponential, Inverted Weibull, Pareto, Gumbel) distributions. The expressions for Bayes estimators of the shape parameters have been derived under four different prior distributions assuming four different loss functions. The posterior predictive distributions have been obtained, and the comparison between estimators made by using the mean squared errors through generated different sample sizes by using simulation technique. In general, the performance of estimators under Chi-square prior using squared error loss function is the best.

**Keyword:** Bayesian Analysis, Exponentiated (Weibull, Exponential, Inverted Weibull, Pareto, Gumbel) distributions, simulation.

### التحليل البيزي لخمس من التوزيعات الأسية تحت دوال لاحقه ودوال خساره مختلفه

ندى صباح كرم\*

قسم الرياضيات، كلية التربية، الجامعة المستنصرية، بغداد، العراق

### الخلاصة

يدرس البحث التحليل اللاحق لخمس من التوزيعات التي تم جعلها اسية (ويبل، اسي، معكوس ويبل، باريتو، غامبل). تم اشتقاق مقدرات بيز لمعلمة الشكل تحت اربع توزيعات سابقة مختلفة بافتراض اربع دوال خساره مختلفه. تم ايجاد التوزيعات اللاحقه، وتمت المقارنه بين المقدرات من خلال دراسة محاكاة عند حجوم عينات وقيم معالم مختلفه باستخدام متوسط مربعات الخطأ. تم التوصل، بشكل عام، الى ان المقدرات تحت التوزيع السابق مربع-كاي باستخدام مربع الخطأ كدالة خساره هي الأفضل انجازا.

### 1. Introduction.

The exponential distribution has been widely used in failure time data analysis and is preferred for situations where hazard rate is constant, in case of monotonic hazard rate, a number of distributions have been suggested but weibull and Gamma distributions are mostly used . [1]

Exponentated distributions can be obtained by three techniques:

1. Powering a positive real number  $\alpha$  to the cumulative distribution function (CDF), i.e. if we have CDF  $F(x)$  of any random variable  $X$ , then the function:  $G(z) = [F(z)]^\alpha$  ,  $\alpha > 0$ , Is called an exponentiated distribution.

2. Using the formula  $G(z) = 1 - [1 - F(z)]^\alpha$  ,  $\alpha > 0$ .

3. Using the transformation  $z = \log(x)$  , where  $x$  is non-negative random variables.

For the same distribution , we can use more than one technique from previous techniques. [2]

The exponentiated exponential distribution (EE) was used during the first half of the nineteenth century as a particular case of Gompertz-Verhulst distribution function, the genesis of the model, several properties and different estimation procedures are discussed by Gupta and Kundu, it was observed that many properties of this new family are quite similar to those of a Weibull or a Gamma family; therefore it can be used as a possible alternative to a Weibull or a Gamma distribution [3 - 5]

Showky and Bakoban discussed the Exponentiated Gamma (EG) distribution as an important model of life time models and derived Bayesian and non- Bayesian estimators of the shape parameter, reliability and failure rate functions in the case of complete and type-II censored samples [6], they addressed the estimation problem of the finite mixture of two components [7]. Singh et al. proposed Bayes estimators of the parameter of the (EG) distribution and associated reliability function under General Entropy loss function for a censored sample [8]. Raja and Mir proposed extension of four different exponentiated distributions, two parameter Exponentiated Weibull (EW) was found to be suitable to fit unimodels, monotonic and risk functions unlike weibull model [9]. Raqab and Madi considered the Bayesian estimation and prediction for the Exponentiated Rayleigh (ER) model using informative and non-informative priors [10].

Flaih, et al., Considered the standard Exponentiated Inverted Weibull distribution (EIW) that generalizes the standard inverted weibull distribution [11].

Person and Ryden discussed the estimation of T-year return values for significant wave height in a case study using Exponentiated Gumbel (EGu) or generalized extreme value distribution. [12]

Hassan and Basheikh deals with the Bayesian and non- Bayesian estimation of reliability of an s-out of-k system with non-identical component strengths using Exponentiated Pareto (EP) distribution [13]. Abdul-Moniem and Abdel- Hamed generalized the Lomax distribution by powering a positive real number to the CDF, this known as Exponentiated Lomax (EL) distribution, some properties are discussed and the parameter estimation handled [2].

In this paper, we compare the Bayesian estimators of the shape exponentiated parameters of five different exponentiated distributions (EW, EE, EIW, EP and EGu) using four different prior distributions (Non-informative, Exponential, Chi-square and Gamma) distributions under four different loss functions (Squared error, Quadratic, Modified linear exponential and Non- linear exponential), the performance of the obtained estimators are compared by using the mean square error, through generated many sample sizes by using simulation technique.

## 2. The Exponentiated Distributions.

### 2-1. The Exponentiated Weibull Distribution.

The cumulative density function (CDF) corresponding to random variable X with EW distribution is given as : [9]

$$F(x; \beta, \lambda, \sigma) = [1 - \exp(-\sigma x)^\lambda]^\beta \quad x > 0$$

where  $\lambda > 0$ ,  $\beta > 0$  are shape parameters and  $\sigma > 0$  is a scale parameter.

It will be a weibull distribution if  $\beta = 1$  and the exponential distribution if  $\lambda = 1$  and  $\beta = 1$ . The great flexibility of this model is in fitting survival data. Now if  $\lambda = 1$  then

$$F(x; \beta, \sigma) = [1 - e^{-\sigma x}]^\beta \quad x > 0; \sigma, \beta, > 0$$

and the PDF is:  $f(x; \beta, \sigma) = \sigma \beta e^{-\sigma x} [1 - e^{-\sigma x}]^{\beta-1} \dots (1)$

The random number X has been generated by inverse function method, which is for uniform random U:  $X = \frac{-\frac{1}{\sigma} \ln(1-U^{\frac{1}{\beta}})}{\lambda}$

### 2-2. The Exponentiated Exponential Distribution.

The EE distribution has the CDF as: [5]

$$F(x; \alpha, \lambda) = (1 - e^{-\lambda x})^\alpha \quad x > 0; \alpha, \lambda > 0$$

where  $\alpha$  is the shape parameter and  $\lambda$  is the scale parameter, if  $\alpha=1$  then it will be exponential distribution with scale parameter  $\lambda$ . Then if  $\lambda = 1$  the CDF will be as:

$$F(x; \alpha) = (1 - e^{-x})^\alpha$$

and then the PDF as :  $f(x; \alpha) = \alpha e^{-x} (1 - e^{-x})^{\alpha-1} \quad x > 0 ; \alpha > 0 \quad \dots(2)$

The random number X has been generated by inverse function method, which is for uniform random U:  $X = -\ln\left(1 - U^{\frac{1}{\alpha}}\right)$

### 2-3. The Exponentiated Inverted Weibull Distribution.

The random variable X has a standard EIW distribution if its CDF takes the following from: [11]

$$F(x; k, \lambda) = [\exp(-x^{-\lambda})]^k \quad x > 0 ; k, \lambda > 0$$

Which is simply the k-th power of the CDF of the standard inverted Weibull distribution. Here , k and  $\lambda$  are the shape parameters. For k=1, it represents the standard IWD, and for  $\lambda = 1$  it represents the Exponentiated Inverted Exponential distribution(EIED). Thus, the EIWD is a generalization of the EIED as well as the IWD. Then if  $\lambda = 1$

$$F(x, k) = [\exp(-x^{-1})]^k = \exp[-kx^{-1}] \quad x > 0 ; k > 0$$

and the PDF is:  $f(x, k) = kx^{-2} (\exp(-kx^{-1})) \quad x > 0 ; k > 0 \quad \dots(3)$

The EIWD also has a physical interpretation as for parallel systems. And the random number X has been generated by inverse function method, which is for uniform random U:  $X = -\left[\ln\left(U^{\frac{1}{k}}\right)\right]^{-1}$

### 2-4. The Exponentiated Pareto Distribution.

A random variable X has EPD if the CDF is:[14]

$$F(x; \theta, \lambda) = [1 - (1+x)^{-\lambda}]^{\theta} \quad x > 0; \theta, \lambda > 0$$

where  $\theta$  and  $\lambda$  are two shape parameters. When  $\theta = 1$ , the above distribution corresponds to the standard Pareto distribution of the second kind. And when  $\lambda = 1$  it will be as:

$$F(x; \theta) = [1 - (1+x)^{-1}]^{\theta}$$

Therefore the PDF is:  $f(x; \theta) = \theta[1 - (1+x)^{-1}]^{\theta-1} (1+x)^{-2}$   
 $= \frac{\theta}{(1+x)^2} \left[\frac{x}{1+x}\right]^{\theta-1} = \frac{\theta x^{\theta-1}}{(1+x)^{\theta+1}} \quad x > 0; \theta > 0 \quad \dots(4)$

and the random number X has been generated by inverse function method, which is for uniform random U:  $X = U^{\frac{1}{\theta}} \left[1 - U^{\frac{1}{\theta}}\right]^{-1}$

### 2-5. The Exponentiated Gumbel Distribution.

The CDF of the EGu distribution is:[15]

$$F(x, Q, \lambda) = \left[\exp\left(-e^{-\frac{x}{\lambda}}\right)\right]^Q \quad -\infty < x < \infty; Q, \lambda > 0$$

which is simply the  $Q^{th}$  power of CDF of the Gumbel distribution. Where Q and  $\lambda$  are shape and scale parameters respectively. Then if  $\lambda = 1$  the CDF will be:

$$F(x; Q) = [\exp(-e^{-x})]^Q$$

and the PDF is:  $f(x; Q) = Qe^{-x} (\exp(-e^{-x}))^{Q-1} \quad -\infty < x < \infty; Q > 0 \quad \dots(5)$

The random number X has been generated by inverse function method, which is for uniform random U:  $X = -\ln\left[-\ln U^{\frac{1}{Q}}\right]$

## 3. Different Bayesian Estimators of the shape parameters.

In this section Bayes estimators of the shape parameter for four different prior functions and four different loss functions has been determined.

### 3-1. Posterior distributions with different priors.

For Bayesian estimation, we specify four different prior distributions for the shape parameter, as:

1- The **Non – informative** prior , for any parameter A, with PDF as:  $g(A) = \frac{1}{A} \quad A > 0$

2- The **Exponential** prior, for any parameter A, as:  $g(A) = be^{-bA} \quad A > 0; b > 0$

3- The **Chi-square** prior is assumed to be:  $g(A) = \frac{1}{\Gamma(\frac{r}{2}) 2^{\frac{r}{2}}} A^{\frac{r}{2}-1} e^{-\frac{A}{2}} \quad A > 0; r > 0$

4- The **Gamma** prior, assumed to be:  $g(A) = \frac{b^a}{\Gamma(a)} A^{a-1} e^{-bA} \quad A > 0; a, b > 0$

where r, a and b are hyper parameters.

Then the posterior density function of the shape parameters for the given random sample X is well known as:

$$f(A, x) = \frac{f(x_i/A) g(A)}{\int_A f(x_i/A) g(A) d(A)}$$

Now the posterior distributions under the assumption of **Non-informative** prior are:

3-1-1. **For EWD**, then by eq. (1):

$$g(\beta) \prod_{i=1}^n f(x_i/\beta) = \frac{1}{\beta} \sigma^n \beta^n \prod_{i=1}^n (1 - e^{-\sigma x_i}) \beta^{-1} e^{-\sigma \sum_{i=1}^n x_i} \\ = \sigma^n \beta^{n-1} \exp[-\beta(-\sum_{i=1}^n \ln(1 - e^{-\sigma x_i}))] \\ \cdot \exp[-\sum_{i=1}^n \ln(1 - e^{-\sigma x_i}) - \sigma \sum_{i=1}^n x_i]$$

$$\text{then } f(\beta/x) = \frac{[-\sum_{i=1}^n \ln(1 - e^{-\sigma x_i})]^n}{\Gamma(n)} \beta^{n-1} e^{-\beta(-\sum_{i=1}^n \ln(1 - e^{-\sigma x_i}))}$$

which implies that:  $f(\beta/x) \sim \text{Gam}(n, -\sum_{i=1}^n \ln(1 - e^{-\sigma x_i}))$

3-1-2. **For EED**, by eq. (2):

$$g(\alpha) \prod_{i=1}^n f(x_i/\alpha) = \frac{1}{\alpha} \alpha^n e^{\sum_{i=1}^n x_i} e^{(\alpha-1)\sum_{i=1}^n \ln(1 - e^{-x_i})} \\ = \alpha^{n-1} \exp[-\alpha(-\sum_{i=1}^n \ln(1 - e^{-x_i}))] \\ \cdot \exp[-\sum_{i=1}^n x_i - \sum_{i=1}^n \ln(1 - e^{-x_i})]$$

$$\text{Then } f(\alpha/x) = \frac{[-\sum_{i=1}^n \ln(1 - e^{-x_i})]^n}{\Gamma(n)} \alpha^{n-1} e^{-\alpha(-\sum_{i=1}^n \ln(1 - e^{-x_i}))}$$

which implies that:  $f(\alpha/x) \sim \text{Gam}(n, -\sum_{i=1}^n \ln(1 - e^{-x_i}))$

3-1-3. **For EIWD**, and by eq. (3):

$$g(k) \prod_{i=1}^n f(x_i/k) = \frac{1}{k} k^n \prod_{i=1}^n x_i^{-2} e^{-k \sum_{i=1}^n x_i^{-1}}$$

$$\text{then } f(k/x) = \frac{[\sum_{i=1}^n x_i^{-1}]^n}{\Gamma(n)} k^{n-1} e^{-k \sum_{i=1}^n x_i^{-1}}$$

which implies that:  $f(k/x) \sim \text{Gam}(n, \sum_{i=1}^n x_i^{-1})$

3-1-4. **For EPD**, and by eq. (4):

$$g(\theta) \prod_{i=1}^n f(x_i/\theta) = \frac{1}{\theta} \theta^n \frac{e^{(\theta-1)\sum_{i=1}^n \ln x_i}}{e^{(\theta+1)\sum_{i=1}^n \ln(1+x_i)}} \\ = \theta^{n-1} e^{-\theta(\sum_{i=1}^n \ln(1+x_i) - \sum_{i=1}^n \ln x_i)} e^{-\sum_{i=1}^n \ln x_i - \sum_{i=1}^n \ln(1+x_i)}$$

$$\text{Then } f(\theta/x) = \frac{[\sum_{i=1}^n \ln(1+x_i) - \sum_{i=1}^n \ln x_i]^n}{\Gamma(n)} \theta^{n-1} e^{-\theta(\sum_{i=1}^n \ln(1+x_i) - \sum_{i=1}^n \ln x_i)}$$

implies that:  $f(\theta/x) \sim \text{Gam}(n, \sum_{i=1}^n \ln(1+x_i) - \sum_{i=1}^n \ln x_i)$

3-1-5. **For EGUD**, and by eq. (5):

$$g(Q) \prod_{i=1}^n f(Q/x_i) = \frac{1}{Q} Q^n e^{-\sum_{i=1}^n x_i} e^{Q \sum_{i=1}^n \ln(\exp(-e^{-x_i}))} = Q^{n-1} e^{-\sum_{i=1}^n x_i} e^{-Q \sum_{i=1}^n e^{-x_i}}$$

$$\text{then } f(Q/x) = \frac{(\sum_{i=1}^n e^{-x_i})^n}{\Gamma(n)} Q^{n-1} e^{-Q \sum_{i=1}^n e^{-x_i}}$$

Which implies that:  $f(Q/x) \sim \text{Gam}(n, \sum_{i=1}^n e^{-x_i})$

By the same procedure, we can drive the posterior distributions under the assumption of exponential, Chi-Square, Gamma prior distributions as recorded in table (1).

### 3-2. Bayes Estimators of the shape parameter for different Loss functions.

The Bayes estimator and associated posterior risks derived under the assumption of prior distributions using the following four different loss functions:

#### 1. Squared Error loss function (SELF).

Here we have determined Bayes estimator of any parameter A for SELF defined as:[16]

$$L(\hat{A}, A) = (\hat{A} - A)^2$$

For SELF the Bayes estimators for the shape parameters of the exponentiated distributions,  $\hat{A}_{BS}$ , are the mean of posterior density functions.

#### 2. Quadratic Loss function (QLF).

Now suppose the loss function is quadratic, which is defined as:[16]  $L(\hat{A}, A) = \left(\frac{\hat{A}-A}{A}\right)^2$

Under QLF the Bayes estimators for the shape parameters are obtained by solving:

$$\frac{\partial}{\partial t} \int_A \left(\frac{\hat{A}-A}{A}\right)^2 f(A/\underline{x}) dA = 0$$

which implies that:  $\hat{A}_{BQ} = \int_A A^{-1} f(A/\underline{x}) dA / \int_A A^{-2} f(A/\underline{x}) dA$

#### 3. Modified liner exponential loss function (MLINEXLF).

Let us consider the MLINEX loss function defined as:[16]

$$L(\hat{A}; A) = w \left[ \left(\frac{\hat{A}}{A}\right)^c - c \ln\left(\frac{\hat{A}}{A}\right) - 1 \right] \quad w > 0, c \neq 0$$

For this loss function Bayes estimator of the shape parameters are obtained from:

$$\hat{A}_{BM} = [E(A^{-c}/\underline{x})]^{-\frac{1}{c}} \quad \text{where} \quad E(A^{-c}/\underline{x}) = \int_A A^{-c} f(A/\underline{x}) dA$$

#### 4. Non-Linear exponential loss function (NLINEXF).

Let us consider the following NLINEX loss function of the form:[16]

$$L(D) = J[\exp(cD) + cD^2 - cD - 1] \quad J > 0, c > 0$$

where D represents the estimation error i.e.  $D = \hat{A} - A$ .

For NLINEX loss function the Bayes estimators of the shape parameters are:

$$\hat{A}_{BM} = \frac{-1}{c+2} [\ln E_A(e^{-cA}/\underline{x}) - 2E_A(A/\underline{x})]$$

Therefore; the Bayes estimator of the shape parameter of the exponentiated distribution under assumption of the Non-informative and Gamma prior distributions are derived and recorded in tables-(2, 3).

Since the exponential and the Chi-square prior distributions are special cases from Gamma prior distribution, then the Bayes estimators can be derived as in table (3) by  $a=1$  for Exponential prior distribution, and by  $a=r/2$  and  $b=1/2$  for Chi-square prior distributi

**Table 1-** posterior distributions under the four prior distributions.

<b>posterior distributions under Non-informative prior distribution.</b>	
EWD	$f(\beta/\underline{x}) \sim \text{Gam}\left(n, -\sum_{i=1}^n \ln(1 - e^{-\sigma x_i})\right)$
EED	$f(\alpha/\underline{x}) \sim \text{Gam}\left(n, -\sum_{i=1}^n \ln(1 - e^{-x_i})\right)$
EIWD	$f(k/\underline{x}) \sim \text{Gam}\left(n, \sum_{i=1}^n x_i^{-1}\right)$
EPD	$f(\theta/\underline{x}) \sim \text{Gam}\left(n, \sum_{i=1}^n \ln(1 + x_i) - \sum_{i=1}^n \ln x_i\right)$
EGuD	$f(\theta/\underline{x}) \sim \text{Gam}\left(n, \sum_{i=1}^n e^{-x_i}\right)$
<b>posterior distributions under Exponential prior distribution.</b>	
EWD	$f(\beta/\underline{x}) \sim \text{Gam}\left(n + 1, b - \sum_{i=1}^n \ln(1 - e^{-\sigma x_i})\right)$
EED	$f(\alpha/\underline{x}) \sim \text{Gam}\left(n + 1, b - \sum_{i=1}^n \ln(1 - e^{-x_i})\right)$
EIWD	$f(k/\underline{x}) \sim \text{Gam}\left(n + 1, b + \sum_{i=1}^n x_i^{-1}\right)$
EPD	$f(\theta/\underline{x}) \sim \text{Gam}\left(n + 1, b - \sum_{i=1}^n \ln x_i + \sum_{i=1}^n \ln(1 + x_i)\right)$
EGuD	$f(\theta/\underline{x}) \sim \text{Gam}\left(n + 1, b + \sum_{i=1}^n e^{-x_i}\right)$
<b>posterior distributions under Chi-Square prior distribution.</b>	
EWD	$f(\beta/\underline{x}) \sim \text{Gam}\left(n + \frac{r}{2}, \frac{1}{2} - \sum_{i=1}^n \ln(1 - e^{-\sigma x_i})\right)$
EED	$f(\alpha/\underline{x}) \sim \text{Gam}\left(n + \frac{r}{2}, \frac{1}{2} - \sum_{i=1}^n \ln(1 - e^{-x_i})\right)$
EIWD	$f(k/\underline{x}) \sim \text{Gam}\left(n + \frac{r}{2}, \frac{1}{2} + \sum_{i=1}^n x_i^{-1}\right)$
EPD	$f(\theta/\underline{x}) \sim \text{Gam}\left(n + \frac{r}{2}, \frac{1}{2} - \sum_{i=1}^n \ln x_i + \sum_{i=1}^n \ln(1 + x_i)\right)$
EGuD	$f(\theta/\underline{x}) \sim \text{Gam}\left(n + \frac{r}{2}, \frac{1}{2} + \sum_{i=1}^n e^{-x_i}\right)$
<b>posterior distributions under Gamma prior distribution.</b>	
EWD	$f(\beta/\underline{x}) \sim \text{Gam}\left(n + a, b - \sum_{i=1}^n \ln(1 - e^{-\sigma x_i})\right)$
EED	$f(\alpha/\underline{x}) \sim \text{Gam}\left(n + a, b - \sum_{i=1}^n \ln(1 - e^{-x_i})\right)$
EIWD	$f(k/\underline{x}) \sim \text{Gam}\left(n + a, b + \sum_{i=1}^n x_i^{-1}\right)$
EPD	$f(\theta/\underline{x}) \sim \text{Gam}\left(n + a, b - \sum_{i=1}^n \ln x_i + \sum_{i=1}^n \ln(1 + x_i)\right)$
E Gu D	$f(\theta/\underline{x}) \sim \text{Gam}\left(n + a, b + \sum_{i=1}^n e^{-x_i}\right)$

**Table 2-** the Bayes estimator under Non-informative prior assumption.

Dist.s		BS	BQ
EWD	$\hat{\beta}$	$\frac{n}{-\sum_{i=1}^n \ln(1 - e^{-\sigma x_i})}$	$\frac{n - 2}{-\sum_{i=1}^n \ln(1 - e^{-\sigma x_i})}$
EED	$\hat{\alpha}$	$\frac{n}{-\sum_{i=1}^n \ln(1 - e^{-x_i})}$	$\frac{n - 2}{-\sum_{i=1}^n \ln(1 - e^{-x_i})}$
EIWD	$\hat{k}$	$\frac{n}{\sum_{i=1}^n x_i^{-1}}$	$\frac{n - 2}{(\sum_{i=1}^n x_i^{-1})}$
EPD	$\hat{\theta}$	$\frac{n}{\sum_{i=1}^n \ln(1 + x_i) - \sum_{i=1}^n \ln x_i}$	$\frac{n - 2}{\sum_{i=1}^n \ln(1 + x_i) - \sum_{i=1}^n \ln x_i}$
EGuD	$\hat{Q}$	$\frac{n}{\sum_{i=1}^n e^{-x_i}}$	$\frac{n - 2}{\sum_{i=1}^n e^{-x_i}}$
		<b>BM</b>	<b>BN</b>
EWD	$\hat{\beta}$	$\left[ \frac{\Gamma(n - c)}{\Gamma n} \right]^{\frac{-1}{c}} \left( -\sum_{i=1}^n \ln(1 - e^{-\sigma x_i}) \right)^{-1}$	$\frac{n}{c + 2} \left[ \ln \left( 1 + \frac{c}{-\sum_{i=1}^n \ln(1 - e^{-\sigma x_i})} \right) + \frac{2}{-\sum_{i=1}^n \ln(1 - e^{-\sigma x_i})} \right]$
EED	$\hat{\alpha}$	$\left[ \frac{\Gamma(n - c)}{\Gamma n} \right]^{\frac{-1}{c}} \left( -\sum_{i=1}^n \ln(1 - e^{-x_i}) \right)^{-1}$	$\frac{n}{c + 2} \left[ \ln \left( 1 + \frac{c}{-\sum_{i=1}^n \ln(1 - e^{-x_i})} \right) + \frac{2}{-\sum_{i=1}^n \ln(1 - e^{-x_i})} \right]$
EIWD	$\hat{k}$	$\left[ \frac{\Gamma(n - c)}{\Gamma n} \right]^{\frac{-1}{c}} \left( \sum_{i=1}^n x_i^{-1} \right)^{-1}$	$\frac{n}{c + 2} \left[ \ln \left( 1 + \frac{c}{(-\sum_{i=1}^n x_i^{-1})} \right) + \frac{2}{(-\sum_{i=1}^n x_i^{-1})} \right]$
EPD	$\hat{\theta}$	$\left[ \frac{\Gamma(n - c)}{\Gamma n} \right]^{\frac{-1}{c}} \left( \sum_{i=1}^n \ln(1 + x_i) - \sum_{i=1}^n \ln x_i \right)^{-1}$	$\frac{n}{c + 2} \left[ \ln \left( 1 + \frac{c}{\sum_{i=1}^n \ln(1 + x_i) - \sum_{i=1}^n \ln x_i} \right) + \frac{2}{\sum_{i=1}^n \ln(1 + x_i) - \sum_{i=1}^n \ln x_i} \right]$
EGuD	$\hat{Q}$	$\left[ \frac{\Gamma(n - c)}{\Gamma n} \right]^{\frac{-1}{c}} \left( \sum_{i=1}^n e^{-x_i} \right)^{-1}$	$\frac{n}{c + 2} \left[ \ln \left( 1 + \frac{c}{\sum_{i=1}^n e^{-x_i}} \right) + \frac{2}{\sum_{i=1}^n e^{-x_i}} \right]$



**Table 3-** the Bayes estimators under Gamma prior distribution.

Dist.s		BS	BQ
EWD	$\hat{\beta}$	$\frac{n+a}{b-\sum_{i=1}^n \ln(1-e^{-\sigma x_i})}$	$\frac{n+a-2}{b-\sum_{i=1}^n \ln(1-e^{-\sigma x_i})}$
EED	$\hat{\alpha}$	$\frac{n+a}{b-\sum_{i=1}^n \ln(1-e^{-x_i})}$	$\frac{n+a-2}{b-\sum_{i=1}^n \ln(1-e^{-x_i})}$
EIWD	$\hat{k}$	$\frac{n+a}{b+\sum_{i=1}^n x_i^{-1}}$	$\frac{n+a-2}{(b+\sum_{i=1}^n x_i^{-1})}$
EPD	$\hat{\theta}$	$\frac{n+a}{b+\sum_{i=1}^n \ln(1+x_i) - \sum_{i=1}^n \ln x_i}$	$\frac{n+a-2}{b+\sum_{i=1}^n \ln(1+x_i) - \sum_{i=1}^n \ln x_i}$
EGuD	$\hat{Q}$	$\frac{n+a}{b+\sum_{i=1}^n e^{-x_i}}$	$\frac{n+a-2}{b+\sum_{i=1}^n e^{-x_i}}$
		<b>BM</b>	<b>BN</b>
EWD	$\hat{\beta}$	$\left[ \frac{\Gamma(n+a-c)}{\Gamma(n+a)} \right]^{-\frac{1}{c}} \left( b - \sum_{i=1}^n \ln(1-e^{-\sigma x_i}) \right)$	$\frac{n+a}{c+2} \left[ \ln \left( 1 + \frac{c}{b - \sum_{i=1}^n \ln(1-e^{-\sigma x_i})} \right) + \frac{2}{b - \sum_{i=1}^n \ln(1-e^{-\sigma x_i})} \right]$
EED	$\hat{\alpha}$	$\left[ \frac{\Gamma(n+a-c)}{\Gamma(n+a)} \right]^{-\frac{1}{c}} \left( b - \sum_{i=1}^n \ln(1-e^{-x_i}) \right)$	$\frac{n+a}{c+2} \left[ \ln \left( 1 + \frac{c}{b - \sum_{i=1}^n \ln(1-e^{-x_i})} \right) + \frac{2}{b - \sum_{i=1}^n \ln(1-e^{-x_i})} \right]$
EIWD	$\hat{k}$	$\left[ \frac{\Gamma(n+a-c)}{\Gamma(n+a)} \right]^{-\frac{1}{c}} \left( b + \sum_{i=1}^n x_i^{-1} \right)^{-1}$	$\frac{n+a}{c+2} \left[ \ln \left( 1 + \frac{c}{b + \sum_{i=1}^n x_i^{-1}} \right) + \frac{2}{b + \sum_{i=1}^n x_i^{-1}} \right]$
EPD	$\hat{\theta}$	$\left[ \frac{\Gamma(n+a-c)}{\Gamma(n+a)} \right]^{-\frac{1}{c}} \left( b + \sum_{i=1}^n \ln(1+x_i) - \sum_{i=1}^n \ln x_i \right)$	$\frac{n+a}{c+2} \left[ \ln \left( 1 + \frac{c}{b + \sum_{i=1}^n \ln(1+x_i) - \sum_{i=1}^n \ln x_i} \right) + \frac{2}{b + \sum_{i=1}^n \ln(1+x_i) - \sum_{i=1}^n \ln x_i} \right]$
EGuD	$\hat{Q}$	$\left[ \frac{\Gamma(n+a-c)}{\Gamma(n+a)} \right]^{-\frac{1}{c}} \left( b + \sum_{i=1}^n e^{-x_i} \right)^{-1}$	$\frac{n+a}{c+2} \left[ \ln \left( 1 + \frac{c}{b + \sum_{i=1}^n e^{-x_i}} \right) + \frac{2}{b + \sum_{i=1}^n e^{-x_i}} \right]$

**4- Simulation results and Conclusions.**

In this section, the results presented of some numerical experiments to compare the performance of the different Bayes estimators for the shape parameters of five considered distributions proposed in the previous sections, applying a Monte Carlo simulations to compare the performance of different estimators, mainly with respect to their mean squared errors(MSEs) for different sample sizes (n=5, 10, 15, 20 and 30) and for two values of the shape parameters(2 and 2.4).The results of MSEs are computed over (1000) replications for two different cases( case 1: a=0.5, b=1, c=2, r=2 and case 2: a=1.5, b=2, c=3, r=3), and recorded in tables from (4) to (8).



**Table 4-** the MSE for Bayes estimators for EWD .

n	prior	$\sigma=1.5, a=1.5, b=2, c=3, r=3$							
		$\beta=2$				$\beta=2.4$			
		BS	BQ	BM	BN	BS	BQ	BM	BN
5	Non	0.7008	1.6449	1.7147	0.9895	4.7947	5.1702	5.1923	4.8270
	Exp	1.0860	1.8431	1.8801	1.3141	4.6969	5.0392	5.0392	4.7297
	Chi	0.4475	1.1031	1.1354	0.6841	4.5464	4.9046	4.9046	4.5857
	Gam	0.9302	1.6338	1.6644	1.1585	4.6132	4.9525	4.9525	4.6484
10	Non	0.7279	0.4588	0.4581	0.3585	3.8747	4.2215	4.2289	3.9354
	Exp	0.2551	0.4990	0.5054	0.3566	3.8533	4.1714	4.1775	3.9104
	Chi	0.6115	0.3602	0.5389	0.2951	3.6638	3.9943	4.0003	3.7295
	Gam	0.2245	0.4199	0.4250	0.3008	3.7757	4.0907	4.0964	3.8349
15	Non	0.4093	0.3098	0.3097	0.2562	3.0288	3.3424	3.3465	3.1132
	Exp	0.2033	0.3244	0.3265	0.2552	3.0745	3.3642	3.3678	3.1515
	Chi	0.3789	0.2686	0.2682	0.2277	2.8565	3.1544	3.1579	2.9429
	Gam	0.1888	0.2847	0.2847	0.2250	3.0041	3.2906	3.2939	3.0826
20	Non	0.2471	0.1998	0.1998	0.1720	2.2852	2.5610	2.5636	2.3851
	Exp	0.1475	0.2199	0.2208	0.1809	2.3827	2.6406	2.6429	2.4734
	Chi	0.2396	0.1807	0.1805	0.1585	2.1493	2.4108	2.4132	2.2491
	Gam	0.1391	0.1960	0.1968	0.1620	2.3203	2.5748	2.5771	2.4120
30	Non	0.1585	0.1373	0.1373	0.1231	1.0664	1.2603	1.2615	1.1741
	Exp	0.1094	0.1423	0.1425	0.1239	1.2201	1.4099	1.4111	1.3209
	Chi	0.1563	0.1287	0.1286	0.1167	0.9938	1.1768	1.1778	1.0983
	Gam	0.1058	0.1312	0.1314	0.1148	1.1749	1.3612	1.3622	1.2753
		$\sigma=1.5, a=0.5, b=1, c=2, r=2$							
5	Non	2.9272	3.3362	3.2389	2.9539	4.6891	5.1042	5.0063	4.7116
	Exp	2.7960	3.1734	3.0821	2.8241	4.5418	4.9318	4.8383	4.5661
	Chi	2.7652	3.1515	3.0580	2.7947	4.5156	4.9135	4.8181	4.5409
	Gam	2.8881	3.2714	3.1795	2.9143	4.6378	5.0318	4.9380	4.6603
10	Non	1.9248	2.2790	2.1903	1.9734	3.6987	4.0745	3.9816	3.7401
	Exp	1.8640	2.1918	2.1095	1.9109	3.6080	3.9619	3.8742	3.6490
	Chi	1.8127	2.1461	2.0623	1.8616	3.5638	3.9239	3.8346	3.6064
	Gam	1.9434	2.2779	2.1942	1.9891	3.6949	4.0529	3.9644	3.7345
15	Non	1.1068	1.3859	1.3144	1.1655	2.7486	3.0859	3.0013	2.8061
	Exp	1.1013	1.3624	1.2955	1.1566	2.7090	3.0277	2.9477	2.7645
	Chi	1.0413	1.3034	1.2362	1.0984	2.6493	2.9723	2.8912	2.7067
	Gam	1.1638	1.4322	1.3636	1.2190	2.7870	3.1102	3.0291	2.8415
20	Non	0.6368	0.8368	0.7851	0.6898	1.1361	1.4329	1.3563	1.2236
	Exp	0.6549	0.8461	0.7969	0.7055	1.1810	1.4642	1.3914	1.2639
	Chi	0.6014	0.7905	0.7412	0.6524	1.0921	1.3734	1.3007	1.1765
	Gam	0.7003	0.8988	0.8474	0.7516	1.2488	1.5401	1.4652	1.3321
30	Non	0.2282	0.3227	0.2972	0.2588	0.3975	0.5466	0.5069	0.4546
	Exp	0.2484	0.3443	0.3186	0.2795	0.4472	0.5988	0.5587	0.5048
	Chi	0.2178	0.3078	0.2835	0.2472	0.3891	0.5327	0.4945	0.4445
	Gam	0.2705	0.3714	0.3445	0.3029	0.4826	0.6408	0.5991	0.5420

**Table 5-** the MSE for Bayes estimators for EED .

n	prior	a=1.5, b=2, c=3, r=3							
		α=2				α=2.4			
		BS	BQ	BM	BN	BS	BQ	BM	BN
5	Non	1.7363	2.5267	2.5770	1.9159	3.5931	4.3982	4.4472	3.7402
	Exp	1.8447	2.4700	2.4986	1.9866	3.5978	4.2619	4.2914	3.7229
	Chi	1.3730	2.0308	2.0582	1.5571	3.1545	3.8725	3.9010	3.3182
	Gam	1.7030	2.3049	2.3294	1.8508	3.4406	4.0905	4.1161	3.5733
10	Non	0.6862	1.1113	1.1215	0.8557	1.8483	2.4524	2.4660	2.0746
	Exp	0.9085	1.3003	1.3083	1.0562	2.1123	2.6397	2.6501	2.2987
	Chi	0.5385	0.8911	0.8982	0.6918	1.5985	2.1316	2.1417	1.8185
	Gam	0.8223	1.1952	1.2025	0.9690	1.9899	2.5022	2.5118	2.1791
15	Non	0.3869	0.2825	0.2823	0.2346	0.5203	0.7637	0.7663	0.6705
	Exp	0.1852	0.3033	0.3054	0.2370	0.7688	1.0325	1.0350	0.9221
	Chi	0.3623	0.2461	0.2457	0.2103	0.4536	0.6693	0.6714	0.5913
	Gam	0.1717	0.2642	0.2659	0.2075	0.7098	0.9624	0.9648	0.8599
20	Non	0.2525	0.2032	0.2032	0.1757	0.2252	0.1954	0.1953	0.1717
	Exp	0.1502	0.2215	0.2224	0.1827	0.1625	0.2253	0.2257	0.2043
	Chi	0.2454	0.1846	0.1844	0.1625	0.2124	0.1794	0.1794	0.1583
	Gam	0.1420	0.1979	0.1986	0.1640	0.1533	0.2057	0.2016	0.1866
30	Non	0.1634	0.1423	0.1423	0.1276	0.9431	1.2240	1.2279	1.0538
	Exp	0.1133	0.1461	0.1466	0.1277	1.0710	1.3326	1.3359	1.1696
	Chi	0.1607	0.1333	0.1332	0.1208	0.8232	1.0766	1.0797	0.9296
	Gam	0.1097	0.1352	0.1355	0.1185	1.0102	1.2644	1.2675	1.1089
		a=0.5, b=1, c=2, r=2							
5	Non	2.8998	3.3186	3.2189	2.9278	4.4145	4.9312	4.8087	4.4495
	Exp	2.7674	3.1530	3.0597	2.7967	4.2501	4.7279	4.6128	4.2869
	Chi	2.7351	3.1300	3.0343	2.7659	4.2096	4.6994	4.5813	4.2483
	Gam	2.8614	3.2533	3.1592	2.8888	4.3672	4.8513	4.7356	4.4013
10	Non	1.9871	3.3333	2.2467	2.0331	2.8366	3.3383	3.2128	2.9162
	Exp	1.9239	2.2450	2.1645	1.9684	2.7742	3.2359	3.1202	2.8499
	Chi	1.8754	2.2019	2.1200	1.9218	2.6900	3.1613	3.0430	2.7697
	Gam	2.0018	2.3291	2.2473	2.0452	2.8862	3.3570	3.2393	2.9600
15	Non	1.1688	1.4463	1.3754	1.2256	1.8327	2.2271	2.1267	1.9241
	Exp	1.1596	1.4194	1.3530	1.2133	1.8420	2.2106	2.1169	1.9279
	Chi	1.1015	1.3626	1.2958	1.1569	1.7481	2.1199	2.0251	1.8371
	Gam	1.2220	1.4886	1.4206	1.2755	1.9309	2.3082	2.2123	2.0161
20	Non	0.2775	0.4307	0.3884	0.3226	0.8555	1.4107	1.0661	0.9465
	Exp	0.3050	0.4590	0.4171	0.3509	0.9151	1.1899	1.1184	1.0020
	Chi	0.2597	0.4023	0.3630	0.3024	0.8226	1.0920	1.0216	0.1900
	Gam	0.3396	0.5040	0.4596	0.3880	0.9801	1.2647	1.1909	1.0681
30	Non	0.1411	0.1258	0.1259	0.1207	0.2356	0.2025	0.2054	0.1934
	Exp	0.1112	0.1168	0.1123	0.1035	0.1722	0.1822	0.1752	0.1606
	Chi	0.1305	0.1173	0.1172	0.1124	0.2080	0.1852	0.1860	0.1748
	Gam	0.1094	0.1094	0.1169	0.1058	0.1702	0.1923	0.1823	0.1654

**Table 6-** the MSE for Bayes estimators for EIWD .

n	prior	a=1.5, b=2, c=3, r=3							
		k=2				k=2.4			
		BS	BQ	BM	BN	BS	BQ	BM	BN
5	Non	1.7990	2.5727	2.6217	1.9703	3.9780	4.6511	4.6916	4.0808
	Exp	1.8922	2.5068	2.5349	2.0287	3.9265	4.4986	4.5238	4.0188
	Chi	1.4358	2.0844	2.1113	1.6129	3.5855	4.1996	4.2237	3.7034
	Gam	1.7523	2.3449	2.3690	1.8947	3.7896	4.3519	4.3739	3.8879
10	Non	0.3305	0.6499	0.6588	0.4626	2.6449	3.1708	3.1824	2.7996
	Exp	0.5479	0.9169	0.9248	0.7057	2.7673	3.2299	3.2389	2.8996
	Chi	0.2493	0.4849	0.4906	0.3510	2.3837	2.8641	2.8730	2.5409
	Gam	0.4737	0.8139	0.8209	0.6243	2.6573	3.1109	3.1192	2.9290
15	Non	0.3492	0.2771	0.2773	0.2262	1.6317	2.0329	2.0383	1.8038
	Exp	0.1918	0.3218	0.3240	0.2514	1.8461	2.2146	2.2192	1.9958
	Chi	0.3238	0.2364	0.2362	0.1987	1.4659	1.8320	1.8365	1.6332
	Gam	0.1748	0.2800	0.2818	0.2195	1.7594	2.1193	2.1236	1.9100
20	Non	0.2466	0.1970	0.1976	0.1702	0.8200	1.1027	1.1055	0.9807
	Exp	0.1453	0.2169	0.2178	0.1782	1.0644	1.3450	1.3476	1.2138
	Chi	0.2395	0.1789	0.1788	0.1572	0.7288	0.9849	0.9872	0.8801
	Gam	0.1371	0.1932	0.1939	0.1595	0.9998	1.2715	1.2740	1.1478
30	Non	0.1679	0.1441	0.1441	0.1298	0.2391	0.2050	0.2049	0.1807
	Exp	0.1141	0.1454	0.1457	0.1272	0.1682	0.2285	0.2290	0.2076
	Chi	0.1657	0.1357	0.1357	0.1234	0.2261	0.1892	0.1891	0.1673
	Gam	0.1110	0.1348	0.1350	0.1184	0.1596	0.2095	0.2099	0.1903
		a=0.5, b=1, c=2, r=2							
5	Non	2.6973	3.1875	3.0701	2.7360	4.2942	4.8546	4.7215	4.3354
	Exp	2.5588	3.0034	2.8952	2.5984	4.1248	4.6395	4.5153	4.1676
	Chi	2.5147	2.9714	2.8601	2.5565	4.0773	4.6058	4.4781	4.1225
	Gam	2.6666	3.1201	3.0106	2.7036	4.2506	4.7729	4.6478	4.2905
10	Non	1.6889	2.0712	1.9748	1.7479	2.9118	3.4037	3.2808	2.9877
	Exp	1.6394	1.9904	1.9018	1.6955	2.8453	3.2989	3.1854	2.9178
	Chi	1.5775	1.9343	1.8440	1.6362	2.7650	3.2279	3.1119	2.8412
	Gam	1.7239	2.0836	1.9930	1.7788	2.9555	3.4176	3.3022	3.0260
15	Non	0.8343	1.1139	1.0413	0.9000	1.7521	2.1496	2.0482	1.8465
	Exp	0.8455	1.1074	1.0395	0.9073	1.7668	2.1381	2.0435	1.8553
	Chi	0.7788	1.0395	0.9714	0.8419	1.6699	2.0440	1.9485	1.7616
	Gam	0.9074	1.1787	1.1086	0.9696	1.8562	2.2367	2.1399	1.9441
20	Non	0.4434	0.6274	0.5782	0.4950	0.7411	1.0175	0.9446	0.8320
	Exp	0.4687	0.6478	0.6005	0.5196	0.8058	1.0740	1.0038	0.8932
	Chi	0.4158	0.5890	0.5430	0.4660	0.7132	0.9741	0.9054	0.8004
	Gam	0.5104	0.6977	0.6484	0.5626	0.8688	1.1478	1.0750	0.9578
30	Non	0.1403	0.1242	0.1246	0.1197	0.2165	0.1921	0.1929	0.1813
	Exp	0.1103	0.1153	0.1109	0.1024	0.1632	0.1795	0.1711	0.1561
	Chi	0.1298	0.1160	0.1160	0.1115	0.1926	0.1774	0.1764	0.1652
	Gam	0.1084	0.1219	0.1154	0.1046	0.1628	0.1911	0.1796	0.1623

**Table 7-** the MSE for Bayes estimators for EPD .

n	prior	a=1.5, b=2, c=3, r=3							
		θ=2				θ=2.4			
		BS	BQ	BM	BN	BS	BQ	BM	BN
5	Non	0.1406	0.1242	0.1242	0.1116	4.2189	4.8065	4.8416	4.2973
	Exp	0.1024	0.1379	0.1382	0.1199	4.1429	4.6523	4.6746	4.2158
	Chi	0.1393	0.1161	0.1160	0.1055	3.8625	4.4059	4.4271	3.9540
	Gam	0.0982	0.1263	0.1265	0.1102	4.0202	4.5221	4.5416	4.0980
10	Non	0.6749	0.4582	0.4584	0.3488	2.5711	3.1059	3.1177	2.7323
	Exp	0.2638	0.5200	0.5265	0.3721	2.7053	3.1750	3.1841	2.8425
	Chi	0.5577	0.3506	0.3500	0.2806	2.3093	2.7965	2.8055	2.4726
	Gam	0.2291	0.4383	0.4437	0.3139	2.5938	3.0540	3.0624	2.7344
15	Non	0.3138	0.2481	0.2484	0.2035	1.3232	1.7272	1.7327	1.5126
	Exp	0.1783	0.3091	0.3112	0.2400	1.5801	1.9560	1.9607	1.7455
	Chi	0.2966	0.2128	0.2127	0.1800	1.1712	1.5353	1.5398	1.3523
	Gam	0.1611	0.2671	0.2689	0.2076	1.4927	1.8581	1.8626	1.6584
20	Non	0.2483	0.2084	0.2084	0.1785	0.6466	0.9127	0.9154	0.8057
	Exp	0.1561	0.2323	0.2323	0.1914	0.8994	1.1731	1.1756	1.0525
	Chi	0.2389	0.1870	0.1869	0.1631	0.5681	0.8066	0.8089	0.7160
	Gam	0.1466	0.2075	0.2083	0.1718	0.8372	1.1009	1.1033	0.9881
30	Non	0.1502	0.1315	0.1315	0.1182	0.2204	0.1925	0.1925	0.1691
	Exp	0.1068	0.1410	0.1413	0.1228	0.1611	0.2245	0.2250	0.2036
	Chi	0.1485	0.1232	0.1231	0.1119	0.2083	0.1766	0.1765	0.1557
	Gam	0.1029	0.1297	0.1299	0.1134	0.1516	0.2048	0.2052	0.1857
		a=0.5, b=1, c=2, r=2							
5	Non	2.7122	3.1972	3.0810	2.7500	4.3090	4.8641	4.7323	4.3494
	Exp	2.5739	3.0143	2.9072	2.6127	4.1402	4.6504	4.5272	4.1822
	Chi	2.5307	2.9830	2.8728	2.5717	4.0935	4.6174	4.4908	4.1379
	Gam	2.6807	3.1298	3.0215	2.7170	4.2649	4.7826	4.6586	4.3014
10	Non	1.7443	2.1204	2.0257	1.8008	2.5121	3.0537	2.9174	2.6083
	Exp	1.6919	2.0377	1.9505	1.7458	2.4696	2.9644	2.8397	2.5598
	Chi	1.6325	1.9841	1.8953	1.6889	2.3685	2.8732	2.7458	2.4636
	Gam	1.7753	2.1293	2.0403	1.8280	2.5890	3.0953	2.9680	2.6772
15	Non	1.0664	1.3462	1.2743	1.1262	1.4528	1.8571	1.7530	1.5576
	Exp	1.0632	1.3249	1.2578	1.1196	1.4885	1.8662	1.7693	1.5863
	Chi	1.0022	1.2646	1.1971	1.0602	1.3812	1.7597	1.6622	1.4822
	Gam	1.1258	1.3951	1.3262	1.1822	1.5788	1.9676	1.8680	1.6764
20	Non	0.6210	0.8201	0.7682	0.6739	0.6885	0.9593	0.8876	0.7788
	Exp	0.6398	0.8304	0.7808	0.6904	0.7550	1.0191	0.9497	0.8422
	Chi	0.5864	0.7740	0.7250	0.6373	0.6630	0.9185	0.8510	0.6495
	Gam	0.6849	0.8823	0.8311	0.7362	0.8168	1.0921	1.0200	0.9059
30	Non	0.1589	0.1360	0.1380	0.1335	0.2266	0.1980	0.1998	0.1877
	Exp	0.1224	0.1222	0.1190	0.1111	0.1678	0.1811	0.1734	0.1581
	Chi	0.1464	0.1265	0.1280	0.1239	0.2005	0.1816	0.1815	0.1701
	Gam	0.1191	0.1275	0.1222	0.1121	0.1666	0.1920	0.1812	0.1640

**Table 8-** the MSE for Bayes estimators for EGuD .

n	prior	a=1.5, b=2, c=3, r=3							
		Q=2				Q=2.4			
		BS	BQ	BM	BN	BS	BQ	BM	BN
5	Non	2.7846	3.2443	3.2719	2.8434	4.3410	4.8845	4.9169	4.4082
	Exp	2.7197	3.1190	3.1366	2.7747	4.2560	4.7320	4.7528	4.3196
	Chi	2.5067	2.9290	2.9455	2.5749	4.0050	4.5110	4.5307	4.3196
	Gam	2.6241	3.0166	3.0319	2.6827	4.1409	4.6106	4.6288	4.2089
10	Non	1.7797	2.1517	2.1598	1.8753	3.0898	3.5373	3.5674	3.2070
	Exp	1.8334	2.1645	2.1709	1.9174	3.1481	3.5643	3.5723	3.2513
	Chi	1.5859	1.9257	1.9320	1.6835	2.8381	3.2721	3.2800	2.9598
	Gam	1.7550	2.0791	2.0850	1.8409	3.0481	3.4578	3.4653	3.1543
15	Non	0.8582	1.1385	1.1424	0.9725	1.8661	2.2592	2.2645	2.0234
	Exp	0.9957	1.2583	1.2616	1.0979	2.0481	2.4079	2.4124	2.1854
	Chi	0.7426	0.9938	0.9909	0.8516	1.6934	2.0552	2.0596	1.8479
	Gam	0.9351	1.1896	1.1927	1.0371	1.9628	2.3151	2.3194	2.1014
20	Non	0.4090	0.5884	0.2902	0.5009	1.0525	1.3472	1.3501	1.2089
	Exp	0.5423	0.7270	0.7288	0.6327	1.2778	1.5623	1.5649	1.4199
	Chi	0.3481	0.5066	0.5081	0.4324	0.9476	1.2174	1.2199	1.0969
	Gam	0.5009	0.6780	0.6796	0.5894	1.2113	1.4884	1.4909	1.3530
30	Non	0.1503	0.1343	0.1343	0.1204	0.2240	0.1930	0.1930	0.1701
	Exp	0.1100	0.1460	0.1463	0.1272	0.1609	0.2229	0.2233	0.2023
	Chi	0.1479	0.1252	0.1252	0.1134	0.2119	0.1778	0.1777	0.1572
	Gam	0.1057	0.1342	0.1345	0.1173	0.1519	0.2035	0.2039	0.1847
		a=0.5, b=1, c=2, r=2							
5	Non	4.2847	4.8486	4.7146	4.3264	1.9574	0.8481	0.8767	0.8818
	Exp	4.1150	4.6326	4.5076	4.1583	0.3594	0.6847	0.5278	0.3240
	Chi	4.0669	4.5985	4.4700	4.1127	0.8851	0.5512	0.5063	0.4599
	Gam	4.2415	4.7668	4.6409	4.2818	0.3574	0.9048	0.6968	0.3969
10	Non	2.5724	3.1071	2.9726	2.6655	0.6695	0.4695	0.4783	0.4275
	Exp	2.5258	3.0149	2.8918	2.6132	0.2918	0.3643	0.3220	0.2544
	Chi	2.4278	2.9269	2.8010	2.5201	0.4703	0.3617	0.3574	0.3195
	Gam	2.6439	3.1440	3.0184	2.7294	0.2849	0.4242	0.3657	0.2761
15	Non	1.6476	2.0479	1.9455	1.7457	0.3425	0.2725	0.2738	0.2534
	Exp	1.6696	2.0435	1.9480	1.7614	0.2072	0.2356	0.2169	0.1849
	Chi	1.5689	1.9450	1.8487	1.6640	0.2842	0.2348	0.2333	0.2157
	Gam	1.7594	2.1431	2.0453	1.8507	0.2024	0.2627	0.2361	0.1946
20	Non	0.8743	1.1598	1.0852	0.9649	0.2611	0.2127	0.2159	0.2048
	Exp	0.9330	1.2080	1.1365	1.0194	0.1776	0.1842	0.1756	0.1577
	Chi	0.8408	1.1106	1.0401	0.9278	0.2288	0.1902	0.1919	0.1821
	Gam	0.9981	1.2828	1.2089	1.0857	0.1722	0.1976	0.1844	0.1613
30	Non	0.2148	0.1872	0.1887	0.1777	0.1468	0.1291	0.1299	0.1250
	Exp	0.1593	0.1732	0.1653	0.1509	0.1151	0.1189	0.1148	0.1064
	Chi	0.1904	0.1724	0.1720	0.1614	0.1358	0.1205	0.1209	0.1164
	Gam	0.1583	0.1841	0.1733	0.1566	0.1129	0.1252	0.1190	0.1083

The respective best performances as in the following tables:

**Table 9-** the best performances for EWD .

prior	$\sigma=1.5, a=1.5, b=2, c=3, r=3$									
	$\beta=2$					$\beta=2.4$				
	n=5	n=10	n=15	n=20	n=30	n=5	n=10	n=15	n=20	n=30
<b>Non</b>	BS	BN	BN	BN	BN	BS	BS	BS	BS	BS
<b>Exp</b>	BS	BS	BS	BS	BS	BS	BS	BN	BN	BN
<b>Chi</b>	BS	BN	BN	BN	BN	BS	BS	BS	BS	BS
<b>Gam</b>	BS	BS	BS	BS	BS	BS	BS	BN	BN	BN
<b>LF</b>										
<b>BS</b>	Chi	Chi	Gam	Gam	Gam	Chi	Chi	Chi	Gam	Chi
<b>BQ</b>	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi
<b>BM</b>	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi
<b>BN</b>	Chi	Chi	Gam	Chi	Gam	Chi	Chi	Chi	Chi	Chi
<b>Performance of Loss fuc.s:</b> BS, BN, BQ, BM <b>Performance of Priors:</b> Chi, Gam, Non, Exp										
prior	$a=0.5, b=1, c=2, r=2$									
	$\beta=2$					$\beta=2.4$				
	n=5	n=10	n=15	n=20	n=30	n=5	n=10	n=15	n=20	n=30
<b>Non</b>	BS	BS	BS	BS	BS	BS	BS	BS	BS	BS
<b>Exp</b>	BS	BS	BS	BS	BS	BS	BS	BS	BS	BS
<b>Chi</b>	BS	BS	BS	BS	BS	BS	BS	BS	BS	BS
<b>Gam</b>	BS	BS	BS	BS	BS	BS	BS	BS	BS	BS
<b>LF</b>										
<b>BS</b>	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi
<b>BQ</b>	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi
<b>BM</b>	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi
<b>BN</b>	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi
<b>Performance of Loss fuc.s:</b> BS, BN, BM, BQ <b>Performance of Priors:</b> Chi, Exp, Non, Gam										

From table (4) it is observed that the values of MSE decreasing convergently with increasing in the sample size for any type of the four prior distributions and loss functions used in this study as the values of  $\beta$  indexed in the table. the achievement as is evident in table (9), the performance of the prior distribution Chi was best for the rest of the distributions, which is no secret that it is a special case of the Gamma distribution, as well as the completion of BS loss function.

**Table 10-** the best performances for EED.

prior	$a=1.5, b=2, c=3, r=3$									
	$\alpha=2$					$\alpha=2.4$				
	n=5	n=10	n=15	n=20	n=30	n=5	n=10	n=15	n=20	n=30
<b>Non</b>	BS	BS	BN	BN	BN	BS	BS	BS	BN	BS
<b>Exp</b>	BS	BN	BS	BS	BS	BS	BS	BS	BS	BS
<b>Chi</b>	BS	BS	BN	BN	BN	BS	BS	BS	BN	BS
<b>Gam</b>	BS	BS	BS	BS	BS	BS	BS	BN	BS	BS
<b>LF</b>										
<b>BS</b>	Chi	Chi	Gam	Gam	Gam	Chi	Chi	Chi	Gam	Chi
<b>BQ</b>	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi
<b>BM</b>	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi
<b>BN</b>	Chi	Chi	Gam	Chi	Gam	Chi	Chi	Chi	Chi	Chi
<b>Performance of Loss F.:</b> BS, BN, BM, BQ <b>Performance of Priors:</b> Chi, Gam, Non, Exp										

prior	a=0.5, b=1, c=2, r=2									
	α =2					α =2.4				
Non	BN	BS	BN	BN	BN	BS	BS	BS	BS	BN
Exp	BS	BS	BS	BS	BS	BS	BS	BS	BS	BN
Chi	BN	BS	BN	BN	BN	BS	BS	BS	BN	BN
Gam	BS	BS	BS	BS	BS	BS	BS	BS	BS	BN
LF										
BS	Chi	Chi	Chi	Chi	Gam	Chi	Chi	Chi	Chi	Gam
BQ	Chi	Chi	Chi	Chi	Gam	Chi	Chi	Chi	Chi	Exp
BM	Chi	Chi	Chi	Chi	Exp	Chi	Chi	Chi	Chi	Exp
BN	Chi	Chi	Chi	Chi	Exp	Chi	Chi	Chi	Chi	Exp
Performance of Loss F.: BS, BN, BQ, BM      Performance of Priors: Chi, Exp, Non, Gam										

In Table (5) illustrated the decreasing values of criterion MSE to the bayse estimators in the study with the increasing the values of sample sizes, and so was the preference of the prior Chi distribution with the advent of all of distrebutions Gam and Exp in second place when changing the value of α, and as is clear in Table (10).

Table 11- the best performances for EIWD.

prior	a=1.5, b=2, c=3, r=3									
	k=2					k =2.4				
	n=5	n=10	n=15	n=20	n=30	n=5	n=10	n=15	n=20	n=30
Non	BS	BS	BN	BN	BN	BS	BS	BS	BS	BN
Exp	BS	BS	BS	BS	BS	BS	BS	BS	BS	BS
Chi	BS	BS	BN	BN	BN	BS	BS	BS	BS	BN
Gam	BS	BS	BS	BS	BS	BS	BS	BS	BS	BS
LF										
BS	Chi	Chi	Gam	Gam	Gam	Chi	Chi	Chi	Chi	Gam
BQ	Chi	Chi	Chi	Chi	Gam	Chi	Chi	Chi	Chi	Chi
BM	Chi	Chi	Chi	Chi	Gam	Chi	Chi	Chi	Chi	Chi
BN	Chi	Chi	Gam	Chi	Gam	Chi	Chi	Chi	Chi	Chi
Performance of Loss F.: BS, BN, BQ, BM      Performance of Priors: Chi, Non, Gam, Exp										
prior	a=0.5, b=1, c=2, r=2									
	k =2					k =2.4				
Non	BS	BS	BS	BS	BN	BS	BS	BS	BS	BN
Exp	BS	BS	BS	BS	BN	BS	BS	BS	BS	BN
Chi	BS	BS	BS	BS	BN	BS	BS	BS	BS	BN
Gam	BS	BS	BS	BS	BN	BS	BS	BS	BS	BN
LF										
BS	Chi	Chi	Chi	Chi	Gam	Chi	Chi	Chi	Chi	Gam
BQ	Chi	Chi	Chi	Chi	Exp	Chi	Chi	Chi	Chi	Chi
BM	Chi	Chi	Chi	Chi	Exp	Chi	Chi	Chi	Chi	Exp
BN	Chi	Chi	Chi	Chi	Exp	Chi	Chi	Chi	Chi	Exp
Performance of Loss F.: BS, BN, BM, BQ      Performance of Priors: Chi, Exp, Non, Gam										

In table (6) also decreases the values of Ms. for this distribution as the increasing of sample sizs, with a Shi prior ranks first in the achievement and the appearance of Non and Exp in second places, as well as stay BS and BN in the first and second places as the best achievement for loss functions. Order achievement has been arranged of this distribution in Table (11).



**Table 12-** the best performances for EPD.

prior	a=1.5, b=2, c=3, r=3									
	θ=2					θ=2.4				
	n=5	n=10	n=15	n=20	n=30	n=5	n=10	n=15	n=20	n=30
<b>Non</b>	BN	BN	BN	BN	BN	BS	BS	BS	BS	BN
<b>Exp</b>	BS	BS	BS	BS	BS	BS	BS	BS	BS	BS
<b>Chi</b>	BN	BN	BN	BN	BN	BS	BS	BS	BS	BN
<b>Gam</b>	BS	BS	BS	BS	BS	BS	BS	BS	BS	BS
<b>LF</b>										
<b>BS</b>	Gam	Gam	Gam	Gam	Gam	Chi	Chi	Chi	Chi	Gam
<b>BQ</b>	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi
<b>BM</b>	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi
<b>BN</b>	Chi	Chi	Gam	Chi	Chi	Chi	Chi	Chi	Chi	Chi
<b>Performance of Loss F.:</b> BS, BN, BQ, BM <b>Performance of Priors:</b> Chi, Non, Gam, Exp										
prior	a=0.5, b=1, c=2, r=2									
	θ=2					θ=2.4				
	n=5	n=10	n=15	n=20	n=30	n=5	n=10	n=15	n=20	n=30
<b>Non</b>	BS	BS	BS	BN	BN	BS	BS	BS	BS	BN
<b>Exp</b>	BS	BS	BS	BS	BN	BS	BS	BS	BS	BS
<b>Chi</b>	BS	BS	BS	BS	BN	BS	BS	BS	BS	BN
<b>Gam</b>	BS	BS	BS	BS	BN	BS	BS	BS	BS	BN
<b>LF</b>										
<b>BS</b>	Chi	Chi	Chi	Chi	Gam	Chi	Chi	Chi	Chi	Gam
<b>BQ</b>	Chi	Exp	Chi	Chi	Exp	Chi	Chi	Chi	Chi	Chi
<b>BM</b>	Chi	Chi	Chi	Chi	Exp	Chi	Chi	Chi	Chi	Chi
<b>BN</b>	Chi	Chi	Chi	Chi	Exp	Chi	Chi	Chi	Chi	Chi
<b>Performance of Loss F.:</b> BS, BN, BM, BQ <b>Performance of Priors:</b> Chi, Non, Exp, Gam										

Similarly, in Table (7), the values of MSE also convergently decreases with increasing of sample sizs, and the preference for priors Chi and Non in the first and second order, and stay BS and BN ranked in the same orders with respect to the loss functions. As is evident in Table (12).

**Table 13-** the best performances for EGuD.

prior	a=1.5, b=2, c=3, r=3									
	Q=2					Q=2.4				
	n=5	n=10	n=15	n=20	n=30	n=5	n=10	n=15	n=20	n=30
<b>Non</b>	BS	BS	BS	BN	BN	BS	BS	BS	BS	BN
<b>Exp</b>	BS	BS	BS	BS	BS	BS	BS	BS	BS	BS
<b>Chi</b>	BS	BS	BS	BN	BN	BS	BS	BS	BS	BN
<b>Gam</b>	BS	BS	BS	BS	BS	BS	BS	BS	BS	BS
<b>LF</b>										
<b>BS</b>	Chi	Chi	Chi	Chi	Gam	Chi	Chi	Chi	Chi	Gam
<b>BQ</b>	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi
<b>BM</b>	Chi	Chi	Chi	Non	Chi	Chi	Chi	Chi	Chi	Chi
<b>BN</b>	Chi	Chi	Chi	Chi	Chi	Gam	Chi	Chi	Chi	Chi
<b>Performance of Loss F.:</b> BS, BN, BQ, BM <b>Performance of Priors:</b> Chi, Gam, Non, Exp										
prior	a=0.5, b=1, c=2, r=2									
	Q=2					Q=2.4				
	n=5	n=10	n=15	n=20	n=30	n=5	n=10	n=15	n=20	n=30
<b>Non</b>	BS	BS	BS	BN	BN	BQ	BN	BN	BN	BN
<b>Exp</b>	BS	BS	BS	BS	BN	BN	BN	BN	BN	BN
<b>Chi</b>	BS	BS	BS	BS	BN	BN	BN	BN	BN	BN
<b>Gam</b>	BS	BS	BS	BS	BN	BS	BN	BN	BN	BN
<b>LF</b>										

<b>BS</b>	Chi	Chi	Chi	Chi	Gam	Gam	Gam	Gam	Gam	Gam
<b>BQ</b>	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Chi	Exp	Exp
<b>BM</b>	Chi	Chi	Chi	Chi	Exp	Chi	Exp	Exp	Exp	Exp
<b>BN</b>	Chi	Chi	Chi	Chi	Exp	Exp	Exp	Exp	Exp	Exp
<b>Performance of Loss F.:</b> BS, BN, BM, BQ <b>Performance of Priors:</b> Chi, Exp, Non, Gam										

Finally, in Table (8), the amount of decreasing is the same with respect to sample sizes. As for the achievement, has remained a priority distribution of Chi in first place while Gam and Exp in the second order for different values of parameters, as in Table (13).

Generally, its observed that the estimates using Chi-square prior under squared error loss function is significantly better than others for all distributions in this study as well as with the other loss functions, and the MSEs of the estimators are decrease as sample sizes are increases.

#### References.

1. Feroze N. and Aslam M.. **2010**. Bayesian Analysis of Exponentiated Gamma Distribution under type II Censored Samples. *Inter. Jou. Of Adv. Sci. and Tech.*, val. 49, December, pp:37-46.
2. Abdul-Moniem I. B. and Abdul-Hameed H.F. **2012**. On Exponentiated Lomax Distribution. *Inter. Jou. Of Math. Archive* -3(5), pp:2144-2150.
3. Kundu D. **2004**. *Exponentiated Exponential Distribution*. Encyclopedia of statistical sciences , 2<sup>nd</sup> edition, Wiley, New York.
4. Gupta R. D. and kundu D. **2001**. Exponentiated Exponential Family: An Alternative to Gamma and weibull distributions. *Biometrical Jou.*, vol. 43, No. 1, pp:117-130.
5. Gupta R. D. and kundu D. **2000**. Generalized Exponential Distribution: Different Method of Estimations. *J. Stut. Comput. Simul.*, vol. 00, pp:1-22.
6. Showky A. I. and Bakoban R.A. **2008**. Bayesian and anaon- Bayesian Estimations of the Exponentiated Gamma Distribution. *App. Math. Sci.*, vol. 2, no. 51, pp:2521-2530.
7. Showky A. I. and Bakoban R.A. **2009**. On infinite mixture of two-component Exponentiated Gamma Distribution. *Jor. Of App. Sci., Res.*, vol. 5, no. 10, pp:1351-1369.
8. Singh S.K., Singh U. and Kumar D. **2011**. Bayesian Estimation of the Exponentiated Gamma Parameter and Reliability function under Asymmetric Loss function. *REVATAT- Stat. Jor.*, vol.9, no. 3, pp:247-260.
9. Raja T.A. and Mir A.H. **2011**. On Extension of some Exponentiated Distributions with Application. *Int. J. Contemp. Math. Sci.*, vol. 6, no. 8, pp:393-400.
10. Raqab M.Z. and Madi M.T. **2009**. Bayesian Analysis for the Exponentiated Rayleigh Distribution. *METRON-Inter. J. of stal.*, vol. 67, no.3, pp:269-288.
11. Flaih A., Elsallaukh H., Mandi E. and Milanova M. **2012**. The Exponentiated Inverted Weibull Distribution. *Appl. Math. Inf. Sci.*, vol. 6, no. 2, pp:167-171.
12. Person K. and Ryden J. **2010** Exponentiated Gumbel Distribution for Estimation of Return Levels of Significant wave height. *Jou. Of En. Stat.*, Vol. 1, no. 3, pp:211-220.
13. Hassan A.S. and Basheikh H. **2012** Reliability Estimation of stress – strength model with Non-identical component strengths: The Exponentiated Pareto case. *Inter. J. of Eng. Res. And App.*, vol. 2, I 3, pp:2774-2781.
14. Showky A. I. and Abu-Zinadah H.H. **2009**. Exponentiated Pareto Distribution : Different Method of Estimations. *Inter. J. contemp. Math. Sciences*, vol. 4, no. 14, 677-693.
15. Kakade C.S., Shirke D.T. and Kundu D. **2008**. Inference for  $p(y < x)$  in Exponentiated Gumbel Distribution. *J. stat. & App.*, vol. 3, no. 1-2, pp:121-133.
16. Rahman H., Roy M.K. and Baizid A.R. **2012**. Bayes Estimation under Conjugate prior for the case of power function Distribution. *Ame. Jou. of math. and stat.*, 2(3), pp:44-48.