Core Polarization Effects on the Inelastic Longitudinal Electron Scattering Form Factors of $^{48,50}\text{Ti}$ and $^{52,54}\text{Cr}$ nuclei

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Abstract
In this paper, inelastic longitudinal electron scattering form factors $C_2$ and $C_4$ transitions have been studied in $^{48,50}\text{Ti}$ and $^{52,54}\text{Cr}$ nuclei with the aid of shell model calculations. The core polarization transition density was evaluated by adopting the shape of Tassie model together with the derived form of the ground state two-body charge density distributions (2BCDD’s). The following transitions have been investigated:

\begin{align*}
2^+_2 \rightarrow 2^+_2 & \quad \text{and} \quad 0^+_3 \rightarrow 4^+_3 \quad \text{of} \quad ^{50}\text{Ti} \\
0^+_3 \rightarrow 2^+_2 & \quad \text{and} \quad 0^+_3 \rightarrow 4^+_2 \quad \text{of} \quad ^{52}\text{Cr} \\
0^+_3 \rightarrow 2^+_2 & \quad \text{and} \quad 0^+_3 \rightarrow 4^+_3 \quad \text{of} \quad ^{54}\text{Cr}
\end{align*}

It is found that the core polarization effects, which represent the collective modes, are essential for reproducing a remarkable agreement between the calculated inelastic longitudinal $C_2$ and $C_4$ form factors and those of experimental data.

Keywords: Inelastic longitudinal form factors, two-body charge density, collective modes

Introduction
The calculations of shell model, carried out within a model space in which the nucleon are restricted to occupy a few orbits are unable to reproduce the measured static moments or transition strengths without scaling factors. Comparison between calculated and measured longitudinal electron

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scattering form factors has long been used as stringent tests of models for transition densities. Various microscopic and macroscopic theories have been used to study excitations in nuclei [1, 2].

Shell model within a restricted model space is one of the models, which succeeded in describing static properties of nuclei, when effective charges are used. Calculations of form factors using the model space wave function alone is inadequate for reproducing the data of electron scattering [3]. Therefore, effects out of the model space, which are called core polarization effects, are necessary to be included in the calculations. Various theoretical methods [4-6] are used for calculations the charge density distributions among them the Hartree-Fock method with the Skyrme effective interaction the theory of finite Fermi systems and the single particle potential method. Comparisons between theoretical and observed longitudinal electron scattering form factors have long been used as stringent test of models of nuclear structure had been studied by Sahu et al. [7]. They calculated longitudinal form factors for some fp-shell nuclei and by the use of Hartree-Fock method, their results are in a good agreement with the experimental data. Core polarization effects can be treated either by connecting the ground state to the J-multipole nω giant resonances [8], where the shape of the transition densities for these excitations is given by Tassie model [9], or by using a microscopic theory [10-13] which permits one particle-hole excitations of the core and also of the model space to describe these longitudinal excitations. In the studies of Massen et al. [14-16], the factor cluster expansion of Clark and co-workers [17] was utilized to derive an explicit form of the elastic charge form factor, truncated at the two-body terms, depends on the harmonic oscillator parameter and the correlation parameter through a Jastrow-type correlation function [18].

The aim of the present work is to study the inelastic longitudinal form factors C2 and C4 and reduced transition probability B(C2) and B(C4) for 48,50Ti and 52,54Cr nuclei. The calculation of form factors using the many particle shell model space alone were known to be inadequate in describing electron scattering data. So effects out of the model space (core-polarization) are necessary to be included in the calculations. The shape of the transition density for the excitation considered in this work was given by the Tassie model [9], where this model is connected with the ground state charge density, where the ground state charge density of the present work is to derive an expression for the ground state two-body charge density distributions (2BCDD’s), based on the use of the two-body wave functions of the harmonic oscillator and the full two-body correlation functions FC’s (which include the tensor correlations TC’s and short range correlations SRC’s). The size parameter b chosen to reproduce the measured ground state root mean square charge radii of these nuclei. The one body density matrix (OBDM) element used in the present work are calculated by adopting the effective interactions GXPF1 [19] and FPD6 [20], by generating the wave functions of a given transition in the known nuclei using the modified version of shell model code OXBASH [21].

Theory

The interaction of the electron with charge distribution of the nucleus gives rise to the longitudinal or Coulomb scattering. The longitudinal form factor is related to the charge density distributions (CDD) through the matrix elements of multiparticle operators $\hat{T}^L_j(q)$ [8].

$$\left| F^L_j(q) \right|^2 = \frac{4\pi}{Z^2(2J_i + 1)} \left\langle f \left| \hat{T}^L_j(q) \right| i \right\rangle^2 \left| F_{cm}(q) \right|^2 \left| F_{fs}(q) \right|^2 \tag{1}$$

Where $Z$ is the atomic number of the nucleus, $F_{cm}(q)$ is the center of mass correction, which remove the spurious state arising from the motion of the center of mass when shell model wave function is used and given by [8].

$$F_{cm}(q) = e^q b^2 / 4A \tag{2}$$

Where $A$ is the nuclear mass number and $b$ is the harmonic oscillator size parameter. The function $F_{fs}(q)$ is the free nucleon form factor and assumed to be the same for protons and neutrons and takes the form [22].

$$F_{fs}(q) = \left[ 1 + \left( \frac{q}{4.33} \right)^2 \right]^{-2} \tag{3}$$

The longitudinal operator is defined as [23].
\[ \hat{T}_{ji}^L(q) = \int dr \, j_j(qr) Y_j^\ast(r)(\Omega) \rho(r,t_z) \]  
(4)

Where \( j_j(qr) \) is the spherical Bessel function, \( Y_j^\ast(\Omega) \) is the spherical harmonic wave function and \( \rho(r,t_z) \) is the charge density operator. The reduced matrix elements in spin and isospin space of the longitudinal operator between the final and initial many particles states of the system including the configuration mixing are given in terms of OBDM elements times the single particle matrix elements of the longitudinal operator [8], i.e.

\[ \langle f \mid \hat{T}_{ji}^L(r_Z,q) \mid i \rangle = \sum_{a,b} OBDM^{JT}(i,f,J,a,b) \left\langle b \right| \hat{T}_{ji}^L(r_Z,q) \left| a \right\rangle \]  
(5)

The many particle reduced matrix elements of the longitudinal operator, consists of two parts one is for the model space and the other is for core polarization matrix element [7].

\[ \langle f \mid \hat{T}_{ji}^{ms}(r_Z,q) \rangle = \langle f \mid \hat{T}_{ji}^{ms}(r_Z,q) \rangle_{ms}^{ms} + \langle f \mid \hat{T}_{ji}^{ms}(r_Z,q) \rangle_{ms}^{core} \]  
(6)

Where the model space element in Eq.(6) has the form [8].

\[ \langle f \mid \hat{T}_{ji}^{ms}(r_Z,q) \rangle_{ms} = \left( \int_0^\infty dr \, j_j(qr) \rho^{ms}_{jz}(i,f,r) \right) \]  
(7)

Where \( \rho^{ms}_{jz}(i,f,r) \) is the transition charge density of model space and given by [8].

\[ \rho^{ms}_{jz}(i,f,r) = \sum_{j^\prime (ms)} OBDM(i,f,J,j^\prime,J^\prime,\tau_z) \left( j \left\| Y_j^\ast \left( j^\prime \right) \right\| R_{nl}(r) R_{n^\prime l^\prime}(r) \right) \]  
(8)

The core polarization matrix element is given by [8].

\[ \langle f \mid \hat{T}_{ji}^{core}(r_Z,q) \rangle_{ms}^{core} = \left( \int_0^\infty dr \, j_j(qr) \rho^{core}_{jz}(i,f,r) \right) \]  
(9)

Where \( \rho_{jz}^{core} \) is the core polarization transition density which depends on the model used for core polarization. To take the core polarization effects into consideration, the model space transition density is added to the core polarization transition density that describes the collective modes of nuclei. The total transition density becomes

\[ \rho_{jz}(i,f,r) = \rho_{jz}^{ms}(i,f,r) + \rho_{jz}^{core}(i,f,r) \]  
(10)

Where \( \rho_{jz}^{core} \) is assumed to have the form of Tassie shape and given by [9].

\[ \rho_{jz}^{core}(i,f,r) = N \frac{1}{2} (1+\tau_z) \left( 1+R_{nl}\right) \left( R_{n^\prime l^\prime} \right) \]  
(11)

Where \( N \) is a proportionality constant. \( \rho_{a}(i,f,r) \) is the ground state charge density distribution. It is derived an effective two-body charge density operator (to be used with uncorrelated wave functions) can be produced by folding the two-body charge density operator with the two-body correlation functions \( \tilde{f}_{ij} \) as [24]

\[ \tilde{\rho}_{ij}^{(2)}(r) = \frac{\sqrt{2}}{2(A-1)} \sum_{i,j} \tilde{f}_{ij} \left\{ \delta \left[ \sqrt{2} \mathbf{r}_i - \mathbf{R}_{ij} - \mathbf{r}_j \right] + \delta \left[ \sqrt{2} \mathbf{r}_i - \mathbf{R}_{ij} + \mathbf{r}_j \right] \right\} \]  
(12)

Where \( \mathbf{r}_{ij} \) and \( \mathbf{R}_{ij} \) of relative and center of mass coordinates and the form of \( \tilde{f}_{ij} \) is given by [25].

\[ \tilde{f}_{ij} = f(r_{ij}) \Delta_1 + f(r_{ij}) \left\{ 1 + \alpha(A) S_{ij} \right\} \Delta_2 \]  
(13)
It is clear that Eq. (13) contains two types of correlations:

1. The two-body short range correlations presented in the first term of Eq. (13) and denoted by 
   \( f(r_{ij}) \). Here \( \Delta_1 \) is a projection operator onto the space of all two-body functions with the exception of \( ^3S_1 \) and \( ^3D_3 \) states. It should be remarked that the short range correlations are central functions of the separation between the pair of particles which reduce the two-body wave function at short distances, where the repulsive core forces the particles apart, and heal to unity at large distance where the interactions are extremely weak. A simple model form of 
   \( f(r_{ij}) \) is given as [25]
   \[
   f(r_{ij}) = \begin{cases} 
   0 & \text{for } r_{ij} \leq r_c \\
   1 - \exp\left\{-\mu (r_{ij} - r_c)\right\} & \text{for } r_{ij} > r_c 
   \end{cases}
   \]
   (14)
   Where \( r_c \) (in fm) is the radius of a suitable hard core and \( \mu = 25 \text{ fm}^{-2} \) [25] is a correlation parameter.

2. The two-body tensor correlations presented in the second term of Eq.(13) are induced by the strong tensor component in the nucleon-nucleon force and they are of longer range. Here \( \Delta_2 \) is a projection operator onto \( ^1S_1 \) and \( ^1D_1 \) states only. \( S_{ij} \) is the usual tensor operator, formed by the scalar product of a second-rank operator in intrinsic spin space and coordinate space and is defined by
   \[
   S_{ij} = \frac{3}{r_{ij}^2} \left( \hat{\sigma}_i \hat{r}_{ij} \hat{\sigma}_j \right) - \hat{\sigma}_i \hat{\sigma}_j
   \]
   (15)
   The parameter \( \alpha (A) \) is the strength of tensor correlations and it is non zero only in the \( ^1S_1 \rightarrow ^1D_1 \) channels.

The Coulomb form factor for this model becomes,

\[
F_{J}^1(q) = \sqrt{\frac{4\pi}{2J_1 + 1}} Z \left[ \int_0^\infty r^2 j_J(qr) \rho^\mu_\sigma (i, f, r) \, dr + \sum_{l=0}^\infty \int_0^\infty r^l r^J j_J(qr) r^{J-1} \frac{d\rho^\mu_\sigma (i, f, r)}{dr} \right] F_{\mu} (q) F_{\sigma} (q)
\]

(16)

The radial integral \( \int_0^\infty dt \ r^{J+1} j_J(qr) \frac{d\rho_o (i, f, r)}{dt} \) can be written as:

\[
\int_0^\infty d\frac{d}{dr} \left\{ r^{J+1} j_J(qr) \rho_o (i, f, r) \right\} dr - \int_0^\infty dr \left( J + 1 \right) r^J j_J(qr) \rho_o (i, f, r)
\]

\[
- \int_0^\infty dr \ r^{J+1} \frac{d}{dr} j_J(qr) \rho_o (i, f, r)
\]

Where the first term gives zero contribution, the second and the third term can be combined together as:

\[
- q \int_0^\infty dr r^{J+1} \rho_o (i, f, r) \left[ \frac{d}{d(qr)} + \frac{J + 1}{qr} \right] j_J(qr)
\]

(17)

From the recursion relation of spherical Bessel function:

\[
\left[ \frac{d}{d(qr)} + \frac{J + 1}{qr} \right] j_J(qr) = j_{J+1}(qr)
\]

(18)

\[
\int_0^\infty d\frac{d}{dr} r^{J+1} j_J(qr) \frac{d\rho_o (i, f, r)}{dr} = - q \int_0^\infty dr r^{J+1} j_{J+1}(qr) \rho_o (i, f, r)
\]

(19)
Hence, the form factor of Eq.(16) takes the form

\[ F_j^l(q) = \left( \frac{4\pi}{2J_i + 1} \right)^{1/2} \frac{1}{Z} \int_0^{\infty} \int_0^{\infty} r^2 j_j(qr) \rho_{J_i}^{MS}(i,f,r) \, dr \, dq \]

\[ \times F_{cm}(q) F_{js}(q) \]  

(20)

The proportionality constant \( N \) can be determined from the form factor evaluated at \( q=k \) (photon point) = \( E_x / hc \) ( \( E_x \) is the excitation energy) i.e., substituting \( q=k \) in Eq.(20), we obtain

\[ N = \int_0^{\infty} \int_0^{\infty} r^2 j_j(kr) \rho_{J_i}^{MS}(i,f,r) - F_j^l(k) \, \frac{\sqrt{2J_i + 1}}{4\pi} \]  

(21)

The reduced transition probability \( B(CJ) \) is written in terms of the form factor in the limit \( q=k \) as [8].

\[ B(CJ) = \left( \frac{2J_i + 1)!}{4\pi k^{2J}} \right) \frac{Z^2 e^2}{(2J_i + 1)} \left| F_j^l(k) \right|^2 \]  

(22)

In Eq. (21), the form factor at the photon point \( (q=k) \) is related to the transition strength \( B(CJ) \). Thus using Eq. (22) in Eq. (21) leads to

\[ N = \int_0^{\infty} \int_0^{\infty} r^2 \rho_{J_i}^{MS}(i,f,r) - \sqrt{(2J_i + 1)}B(CJ) \frac{(2J_i + 1)!}{4\pi k^{2J}} \int_0^{\infty} \int_0^{\infty} r^2 \rho_{J_i}^{MS}(i,f,r) \]  

(23)

the proportionality constant \( N \) can be determined by introducing the reduced transition probability \( B(CJ) \) (eq. 22) into eq.(23).

**Results and Discussion**

The inelastic longitudinal electron scattering form factors C2 and C4 are calculated using an expression for the transition charge density of Eq.(10). The model space transition density is obtained using Eq.(8), where the required OBDM elements was calculated using the OXBASH code [21]. For considering the collective modes of the nuclei, the core polarization transition density of Eq.(11) was evaluated by adopting the Tassie model together with the calculated ground state 2BCDD's of Eq.(12).

All parameters required in the following calculations of 2BCDD's, \( \langle r^2 \rangle^{1/2} \), \( B(CJ) \) and longitudinal \( F(q) \)'s are presented in Table-1.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>( b ) (fm)</th>
<th>( r_c ) (fm)</th>
<th>( \alpha ) (Å)</th>
<th>( \langle r^2 \rangle^{1/2} ) ( _{\text{Theo.}} ) (fm)</th>
<th>( \langle r^2 \rangle^{1/2} ) ( _{\text{Exp.}} ) (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{48}\text{Ti} )</td>
<td>1.920</td>
<td>0.55</td>
<td>0.077</td>
<td>3.612</td>
<td>3.713(15)</td>
</tr>
<tr>
<td>( ^{50}\text{Ti} )</td>
<td>1.932</td>
<td>0.55</td>
<td>0.079</td>
<td>3.393</td>
<td>3.573(2)</td>
</tr>
<tr>
<td>( ^{52}\text{Cr} )</td>
<td>1.941</td>
<td>0.55</td>
<td>0.086</td>
<td>3.476</td>
<td>3.613(17)</td>
</tr>
<tr>
<td>( ^{54}\text{Cr} )</td>
<td>1.960</td>
<td>0.55</td>
<td>0.089</td>
<td>3.511</td>
<td>3.673(14)</td>
</tr>
</tbody>
</table>

The nucleus \( ^{48}\text{Ti} \)

The structure and properties of \( ^{48}\text{Ti} \) are experimentally and theoretically well studied. According to the conventional 1f 2p-shell model, this nucleus is described taking the core at \( ^{40}\text{Ca} \) with eight valence nucleons distributed in 1f_{5/2}, 2p_{3/2}, 1f_{7/2} and 2p_{1/2} shell space. The transitions under investigation are C2(J^T=0^+ → 2^+) 0.893MeV and C4 (J^T=0^+ → 4^+) 1.960 MeV.
The 0.983 MeV (2\(^+\) 2) state

The nucleus is excited from the ground state (0\(^+\) 2) to the excited state (2\(^+\) 2) with an excitation energy 0.983 MeV. Figure-1 shows the relation between the longitudinal Coulomb C2 electron scattering form factors as a function of momentum transfers \(q\). The dashed curves represent the contribution of the model space, the solid curves represent the total contribution, which is obtained by taking the model space together with the core polarization effects by using Tassie model, where the effects of two-body SRC's and TC's are considered, and the dotted symbols represent the experimental data which are taken from [7]. These curves illustrate that the model space is not able to give a satisfactory description with the experimental data for the region of momentum transfer, but once the core polarization effect is added to the model space, the obtained results for the longitudinal C2 form factors become resonable accordance with those of experimental data throughout the whole range of momentum transfer \(q\) as seen in the solid curves of these figure. The calculation of \(B(C2 \uparrow)\) with 1f 2p-shell model is found to be 453 \(e^2\text{fm}^4\), while with 1f 2p-shell model + core polarization effects is 635 \(e^2\text{fm}^4\) in comparison with the measured value 720 \(\pm 40\) \(e^2\text{fm}^4\) [27].

The 1.967 MeV (4\(^+\) 2) state

The nucleus is excited from the ground state (0\(^+\) 2) to the excited state (4\(^+\) 2) with an excitation energy 1.967 MeV. Figure-2 shows the relation between the longitudinal Coulomb C4 electron scattering form factors as a function of momentum transfers (\(q\)). The dashed curve represents the results of the model space (1f 2p-shell), while the solid curves represent the total contribution, which is obtained by taking the model space together with the core polarization effects by using Tassie model, where the effect of two-body SRC's and TC's are considered, and the dotted symbols represent the experimental data [28]. The 1f 2p shell model fail to describe the experimental data and the inclusion of core polarization effects enhances the calculations (solid curve). While the core-polarization effect calculations raise the 1f 2p shell model space calculation making the total theoretical form factor agreed with the experimental values. The calculation of \(B(C4 \uparrow)\) with(1f 2p-shell model) is found to be 435 \(e^2\text{fm}^8\), while with 1f 2p shell model + core polarization effects is 1260 \(e^2\text{fm}^8\) in comparison with the other work 1610 \(e^2\text{fm}^8\) [28].
The nucleus $^{50}$Ti

$^{50}$Ti nucleus has 10 nucleons outside the core $^{40}$Ca and it is possible to perform shell-model calculations for this nucleus in $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ shell space. Two states have been studied in this nucleus, which are: C2 at $E_x=1.550$ MeV and C4 at $E_x=2.675$ MeV.

**The 1.550 MeV (2$^+$) state**

Figure 3 displays the calculations of the C2 form factors for the transition state $J^T=2^+$, $T=3$ at $E_x=1.550$ MeV in $^{50}$Ti. The dashed curves represent the contribution of the model space where the configuration mixing is taken into account, the solid curves represent the total contribution, which is obtained by taking the model space together with the core polarization effects by using Tassie model, where the effect of two-body SRC's and TC's are considered, and the dotted symbols represent the experimental data which are taken from [7]. The model space calculations fail to reproduce the form factors in all momentum transfer regions. The solid curve enhance the C2 form factors in all regions of momentum transfer, and give good agreement with experimental data. The calculation of $B(C \, 2 \uparrow)$ with 1f 2p- shell model is found to be $125.3 \, e^2\text{fm}^4$, while with 1f 2p- shell model + core polarization effects is $233.5 \, e^2\text{fm}^4$ in comparison with the measured value $330 \pm 40 \, e^2\text{fm}^4$ [28].

Figure 2- Inelastic longitudinal form factors for the transition to the $4^+$ state in the $^{48}$Ti with and without core-polarization effects, the experimental data are taken from Ref.[28].

Figure 3- Inelastic longitudinal form factors for the transition to the $2^+$ state in the $^{50}$Ti with and without core-polarization effects, the experimental data are taken from Ref.[7].
The (4' 3) state at 2.675 MeV

The C4 form factors for the transition from the ground state to the state \( J^\pi = 4'^-, T=3 \) at \( E_x = 2.675 \text{MeV} \) is shown in Figure-4. The model space results fail to reproduce the data in the form factors as shown by dashed curve, the solid curves represent the total contribution, which is obtained by taking the model space together with the core polarization effects by using Tassie model, where the effect of two-body SRC's and TC's are considered, and the dotted symbols represent the experimental data. The inclusion of core-polarization effects (solid curve) enhance the C4 form factors. The results of the fp-shell model space with core-polarization effects by using Tassie model give good agreement with the experimental data which are taken from. The calculation of \( B(C4 \uparrow) \) with 1f 2p-shell model is found to be 530 \( e^2 \text{fm}^8 \), while with 1f 2p shell model + core polarization effects is 1980 \( e^2 \text{fm}^8 \) in comparison with the other work 2360 \( e^2 \text{fm}^8 \) [28].

![Figure 4](image)

**Figure 4:** Inelastic longitudinal form factors for the transition to the 4' state in the 50Ti with and without core-polarization effects, the experimental data are taken from Ref. [28].

The nucleus 52Cr

Chromium 52Cr is considered as 40Ca which is treated as a closed shell (core) and 12 nucleons are distributed over 1f 2p-shell model space. Two states have been studied in this nucleus, which are: C2 at \( E_x = 1.430 \text{MeV} \) and C4 at \( E_x = 2.370 \text{MeV} \).

**The (2' 2) state at 1.430 MeV**

Figure-5 displays the calculations of the C2 form factors for the transition state \( J^\pi = 2'^-, T=3 \) at \( E_x = 1.430 \text{MeV} \) in 52Cr. The dashed curve represents the results of the model space (1f 2p-shell), while the solid curve represents the results of 1f 2p-shell with the inclusion of core polarization effects. The 1f 2p-shell model fail to describe the data in both the transition strength and the form factors. The solid curve enhance the C2 form factors in all regions of momentum transfer, and give good agreement with experimental data which are taken from [7]. The calculation of \( B(C2 \uparrow) \) with 1f 2p-shell model is found to be 165.3 \( e^2 \text{fm}^4 \), while with 1f 2p shell+core polarization effect is 549 \( e^2 \text{fm}^4 \) in comparison with the measured value 660±30 \( e^2 \text{fm}^4 \) [27].

**The (4' 2) state at 2.370 MeV**

The C4 form factors for the transition from the ground state to the state \( J^\pi = 4'^-, T=3 \) of nucleus 52Cr with inclusion of core polarization effects as shown by solid curve and that without core polarization effects as shown by dashed curve. The 1f 2p-shell model space underestimated for the first and second maxima as shown by Figure-6. The agreement between the experimental data which are taken from [28] and the results of 1f 2p-shell model with the inclusion of core polarization effects in all region of q.
The nucleus $^{54}$Cr

Chromium $^{54}$Cr has been extensively studied both theoretically and experimentally. For the conventional many particle shell model, this nucleus is considered as an inert $^{48}$Ca core plus six nucleons distributed over 1f 2p- space. The transitions under investigation are C2 ($J^πT=0^+ 3$ to $2^+ 3$), 0.840MeV and C4 ($J^πT=0^+ 3$ to $4^+ 3$), 1.820 MeV.

The (2$^+$ 3) state at 0.840 MeV

The form factors for C2 transition in $^{54}$Cr with an excitation energy 0.840 MeV. The model space fail to describe the form factors in all momentum transfers. A good fit to the C2 data is obtained with the 1f 2p -shell model calculations including core polarization effects by solid curve. In all regions of momentum transfer the form factors is predicted very well in shape as shown by Figure-7. The model space calculations give the value 116 e$^2$ fm$^4$ for the $B(C2 \uparrow)$ which is low in comparison with the measured value 870±40 e$^2$ fm$^4$ [27]. Core-polarization effects enhance the transition strength. In this case the calculated $B(C2 \uparrow)$ values are 703 e$^2$ fm$^4$, which is a good agreement with the measured value.
The (4+ 3) state at 1.820 MeV

In this state the electron excites the nucleus from the ground state \((0^+ 3)\) to the \((4^+ 3)\) excited state. Two peaks where observed as shown in Figure-8 with and without core polarization effect using 1f 2P-shell model space calculation fail to describe the data in all the region of momentum transfers, so the inclusion of core polarization effects leads to an enhancement in the form factors. It is seen that the present calculations are successful in reproducing the magnitude of the form factors at the first and second maximum with including of core polarization effects as shown in Figure-8 by solid curve. The calculation of \(B(C4 \uparrow)\) with (1f 2p-shell model) is found to be \(6454 \, e^2 \cdot fm^8\), while with 1f 2p shell model + core polarization effects is \(116740 \, e^2 \cdot fm^8\) in comparison with other value \(162\times10^3 \, e^2 \cdot fm^8\) [7].
Conclusions
1. The effect of FC’s is, generally, essential in getting good agreement between the calculated results of \( r^2 \) and those of experimental data.
2. The fp-shell models, which can describe the static properties and energy levels, are less successful for describing dynamics properties such as C2 and C4 transitions rates and electron scattering form factors.
3. The core-polarization effect enhances the form factors and makes the theoretical results of the longitudinal form factors closer to the experimental data in the C2 and C4 transitions which are studied in the present work.
4. The inclusion of the core-polarization effects enhance the calculated values of \( B(\text{C2}) \) and \( B(\text{C4}) \) compared with fp-shell model calculation. The theoretical results a good agreement as compared with the experimental data.

References


