On Intuitionistic Fuzzy bi-Ideal With respect To an Element of a near Ring

Showq Mohammed Ebrahim*
Department of Mathematics, College of Education for Girls, AL Kufa University, Al-Kufa, Iraq

Abstract
In this paper we introduce the notions of bi-ideal with respect to an element r denoted by \((r-bi-ideal)\) of a near ring, and the notion fuzzy bi-ideal with respect to an element of a near ring and the relation between F-r-bi-ideal and r-bi-ideal of the near ring, we studied the image and inverse image of r-bi-ideal under epimorphism, the intersection of r-bi-ideals and the relation between this ideal and the quasi ideal of a near ring, also we studied the notion intuitionistic fuzzy bi-ideal with respect to an element r of the near ring \(N\), and give some theorem about this ideal.

Keywords: Near ring, ideal of a near ring, bi-ideal, quasi-ideal, P-regular of near ring, intuitionistic fuzzy, intuitionistic fuzzy bi-ideal.

Introduction
The notion of near ring is first define by G. Pliz [1] in 1983, the notion of bi-ideal interfused by N. Ganesan [2], in 1986 the notion intuitionistis fuzzy sets denoted by K.T. Atanassov [3], in 1987 T.T. Chelvam, N. Ganesan denoted the notion bi-ideals of near-rings, in 1997 the notion Fuzzy Ideal denoted by D.T.K, and Biswas[4], in 2012 the notion P-regular near ring denoted by Aphisit in [5].

1. Preliminaries
In this section we give some concepts that we need.

Definition (1.1) [1]
A left near ring is a set \(N\) together with two binary operations “+” and “.” such that
(1) \((N,+)\) is a group (not necessarily abelian)
(2) \((N,.)\) is a semigroup.
(3) \((n_1 + n_2) \cdot n_3 = n_1 \cdot n_3 + n_2 \cdot n_3\)
For all \(n_1, n_2, n_3, \in N\),

Definition (1.2) [6]:
Let \(N\) be a near ring. A normal subgroup \(I\) of \((N,+)\) is called a left ideal of \(N\) if

*Email: mshowq@yahoo.co.uk
(1) \( I \cdot N \subseteq I \).
(2) \( \forall n, n_i \in N \) and for all \( i \in I \), \( (n_i + i) \cdot n - n_i \in I \)

**Definition (1.3)** \([7]\)

Let \((N_1,+,\cdot)\) and \((N_2,+,\cdot)\) be two near rings. The mapping \( f : N_1 \to N_2 \) is called a near ring homomorphism if for all \( m, n \in N_1 \)

\[
f(m + n) = f(m) + f(n) \quad f(m \cdot n) = f(m) \cdot f(n).
\]

**Theorem (1.4)** \([5]\)

Let \( f : (N_1,+,\cdot) \to (N_2,+,\cdot) \) be a near ring homomorphism.

1. If \( I \) is an ideal of a near ring \( N_1 \), then \( f(I) \) is an ideal of \( N_2 \).
2. If \( J \) is an ideal of a near ring \( N_2 \), then \( f^{-1}(J) \) is an ideal of the near ring \( N_1 \).

**Definition (1.5)** \([8]\)

A nonempty subset \( Q \) of a near ring \( N \) is called a quasi ideal of \( N \) if

1. \( Q \) together with addition is a subgroup of \( N \).
2. \( Q \subseteq Q \cap N \).

**Definition (1.6)** \([2]\)

A nonempty subset \( B \) of a near ring \( N \) is called a bi-ideal of \( N \) if

1. \( B \) together with addition is a subgroup of \( N \).
2. \( B \subseteq B \cap B \).

**Definition (1.7)** \([9]\)

Let \( N \) be a near ring with unity and \( P \) an ideal of \( N \). Then \( N \) is said to be \( P \)-regular near ring if for each \( x \in N \), there exists \( y \in N \) such that \( xy - x \in P \).

**Definition (1.8)** \([10]\)

A near ring \( N \) is called a distributive near ring if

\[
ac + bc = ab + ac \quad \text{for all} \quad a, b, c \in N.
\]

**Theorem (1.9)** \([11]\)

Let \( N \) be a \( P \)-regular near ring. Then for each \( n \in N \), there exists \( n' \in N \) such that \( n' n \in P \).

**Theorem (1.10)** \([11]\)

Let \( N \) be a \( P \)-regular distributive near ring. Then for every left ideal \( L \) and right ideal \( R \) of \( N \),

\[
(P + R) \cap (P + L) = P + RL.
\]

**Definition (1.11)** \([2]\)

Let \( N \) be a non-empty set. A mapping \( \mu : N \to [0,1] \) is called a fuzzy subset of \( N \), where \([0,1]\) is a closed interval of real numbers.

**Definition (1.12)** \([4]\)

Let \( \mu \) be a non-empty fuzzy subset of a near ring \( N \), that is \( \mu(y) \neq 0 \) for some \( y \in N \) then \( \mu \) is said to be fuzzy ideal of \( N \) if it satisfies the following conditions:

1. \( \mu(z - y) \geq \min\{\mu(z), \mu(y)\} \);
2. \( \mu(z + y) \geq \min\{\mu(z), \mu(y)\} \);
3. \( \mu(y + z - y) \geq \mu(z) \);
4. \( \mu(z, y) \geq \mu(y), \forall y, z \in N \).

When the subset of \( N \) satisfies 1, 2 is called fuzzy sub near ring.

**Definition (1.13)** \([3]\)

An intuitionistic fuzzy set \( A \) in a non-empty set \( X \) is an object having the form

\[
A = \{(x, \mu_A(x), \lambda_A(x))| x \in X \},
\]

where the functions \( \mu_A : X \to [0,1] \) and \( \lambda_A : X \to [0,1] \) denote the degree of membership and the degree of non-membership of each element \( x \in X \) to the set \( A \), respectively, and \( 0 \leq \mu_A(x) + \lambda_A(x) \leq 1 \) for all \( x \in X \).
Definition (1.14)[3]
An intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ in $N$ is an intuitionistic fuzzy subnear ring of $N$ if for $x, y \in N$

1. $\mu_A(x - y) \geq \min \{\mu_A(x), \mu_A(y)\}$
2. $\mu_A(xyz) \geq \min \{\mu_A(x), \mu_A(z)\}$
3. $\lambda_A(x - y) \leq \max \{\lambda_A(x), \lambda_A(y)\}$
4. $\lambda_A(xyz) \leq \max \{\lambda_A(x), \lambda_A(z)\}$

Proposition (1.15)[1]
Let $X$ be a non-empty set. A mapping $\mu : N \rightarrow [0,1]$ is a fuzzy set in $X$, the complement of $\mu$, denoted by $\mu^c$, is the fuzzy set in $X$ given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in N$ for any $I \subseteq X$, $X_I$ denotes the characteristic function of $I$.

For any fuzzy set $\mu$ and $h \in [0,1]$, we define two sets, $\bigcup(\mu, h) = \{x \in X | \mu(x) \geq h\}$ and $L(\mu, h) = \{x \in X | \mu(x) \leq h\}$ Which are called upper and lower $h$-level cut of $\mu$ respectively, and can be used to characterize $\mu$.

2. bi-ideal with respect to an element of a near ring $N$

In this section we devoted to study bi-ideal with respect to an element $r$ of a near ring $N$ and give some properties, theorem about this ideal.

Definition (2.1)
A nonempty subset $B$ of a near ring $N$ is called a bi-ideal with respect to an element $r$ of $N$ and denoted by $r$-bi-ideal of a near ring if
1. $B$ together with addition is a subgroup of $N$.
2. $r.BNB \subseteq r.B$, $r \in N$.

Example (2.2).
Let $N = \{0, a, b, c\}$ be the near ring defined by Cayley

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>A</td>
<td>b</td>
<td>c</td>
<td>a</td>
</tr>
<tr>
<td>a</td>
<td>A</td>
<td>0</td>
<td>c</td>
<td>b</td>
</tr>
<tr>
<td>b</td>
<td>B</td>
<td>C</td>
<td>0</td>
<td>a</td>
</tr>
<tr>
<td>c</td>
<td>C</td>
<td>B</td>
<td>a</td>
<td>0</td>
</tr>
</tbody>
</table>

Then $B = \{0, a\}$ is $b$-bi-ideal since $b.BNB \subseteq b.B$.

Remark (2.3)
If $B_1$ and $B_2$ be two $r$-bi-ideal of near ring $N$, then $B_1, B_2$ of $N$ may be not $r$-bi-ideal.

Example (2.4)
Consider the near ring $N = \{0, a, b, c\}$ with addition and multiplication defined by the following tables.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Let $B_1 = \{0, 3\}$ and $B_2 = \{1, 2, 3\}$ are two $3$-bi-ideal of $N$ but $B_1 \cap B_2 = \{3\}$ is not two $3$-bi-ideal of $N$.

Remark (2.5)
Not all $r$-bi-ideal of a near ring are bi-ideal of $N$.

Example (2.6)
Consider the near ring $N$ in example (2.2) let $B = \{0, b\}$ be $b$-bi-ideal but $B$ is not bi-ideal of $N$ since $BNB \not\subseteq B$.
Theorem (2.7)
Let \((N_1,+,..,')\) and \((N_2,+,..,')\) be two near rings, \(f : N_1 \rightarrow N_2\) be an epimorphism and \(B\) be a \(r\)-bi-ideal of \(N_1\). Then \(f(B)\) is a \(r\)-bi-ideal of \(N_2\).

Proof
Let \(B\) be a \(r\)-bi-ideal of \(N_1\), \(f(B)\) is a subgroup of \(N_2\) to prove \(f(B)\) is an \(r\)-bi-ideal of \(N_2\).

\[
f(B) \subseteq rB \text{ Since } B \text{ is an } r\text{-bi-ideal of } N_1
\]
\[
f(r).f(B) = f(r).f(N_1) = f(r).f(B)
\]
\[
f(r).f(B) \subseteq f(r).f(B)
\]
\[
f(B) \text{ is } f(r)\text{-bi-ideal of } N_2
\]

Theorem (2.8)
Let \((N_1,+,..,')\) and \((N_2,+,..,')\) be two near rings, and \(f : N_1 \rightarrow N_2\) be an epimorphism and \(J\) be a \(f(r)\)-bi-ideal of \(N_2\). Then \(f^{-1}(J)\) is a \(r\)-bi-ideal of \(N_1\), where \(y = f(r)\), \(\ker f \subseteq f^{-1}(J)\).

Proof
Let \(r \in N_1\), \(f^{-1}(J)\) is a subgroup of \(N_1\)
\[
y'.J N_2^2 \subseteq y'.J
\]
\[
f^{-1}(y'.J N_2^2) = f^{-1}(y'.J)
\]
\[
f^{-1}(y).f^{-1}(J)f^{-1}(N_2)f^{-1}(J) \subseteq f^{-1}(y).f^{-1}(J)
\]
\[
r.f^{-1}(J)N_2^2 \subseteq r.f^{-1}(J)
\]
\[
J \text{ is a r-bi-ideal of } N_1
\]

Remark (2.9)
Not all \(r\)-bi-ideals of the near ring \(N\) are quasi ideal.

Example (2.10)
Let \(N = \{0,a,b,c\}\) be the near ring defined by caleys

<table>
<thead>
<tr>
<th>+</th>
<th>0</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>a</td>
<td>A</td>
<td>0</td>
<td>c</td>
<td>b</td>
</tr>
<tr>
<td>b</td>
<td>B</td>
<td>C</td>
<td>0</td>
<td>a</td>
</tr>
<tr>
<td>c</td>
<td>C</td>
<td>B</td>
<td>a</td>
<td>0</td>
</tr>
</tbody>
</table>

Let \(B = \{0,a\}\) is \(c\)-bi-ideal since \(c.BNB \subseteq c.B\) but is not quasi ideal.

Theorem (2.11)
Let \(N\) be a \(P\)-regular near ring and \(B\) a \(r\)-bi-ideal of \(N\). Then every \(y \in B\) there exist \(p' \in P\) and \(b' \in B\) such that \(y = p' + b'\).

Proof
Let \(N\) be a \(P\)-regular near ring and \(B\) a \(r\)-bi-ideal of \(N\) and \(y \in B \subseteq N\), there exists \(z \in N\) such that \(r(zy) - y = p\) for some \(p \in P\) thus \(y = -p + r(zy)\) since \(B\) is \(r\)-bi-ideal of \(N\) we have \(r(zy) \in rBNB \subseteq rB\) since \(P \in P\) together with addition is a subgroup of \(N\) we have \(-p \in P\) put \(p' = -p\) and \(b' = r(zy)\) thus \(y = -p + r(zy) = p' + b' \in P + B\).

Theorem (2.12)
Let \(N\) be a \(P\)-regular distributive near ring and let \(B_1, B_2\) are \(r\)-bi-ideals of \(N\). if \(b \in B_1 \cap B_2\) and \(y \in N\), then the element \(b\) can be represented as \(b = p + rh_1 y_1 b_2\) and \(b_1 y_1 b_2 y P \subseteq P\) for some \(p \in P\), \(r, y_i \in N, b_i \in B_i\) and \(b_2 \in B_2\).
Proof
Let \( b \in B_1 \cap B_2 \) since \( N \) is a \( P \)-regular near ring there exists \( y_1 \in N \) such that \( rby_1b - b \in P \) since \( b \in B_1 \cap B_2 \) by theorem (1.9) we have \( b = p_1 + b_1 \) for some \( p_1 \in P \) and \( b_1 \in B \). \( b = p_2 + b_2 \) for some \( p_2 \in P \) and \( b_2 \in B \) since \( rby_1b - b \in P \), we have \( rby_1b - b = p_3 \) for some \( p_3 \in P \) Thus \( b = -p_3 + rby_1b \). Hence
\[
= -p_3 + r(p_1 + b_1)y_1(p_2 + b_2) \\
= -p_3 + rp_1y_1p_2 + rp_1y_1b_2 + rh_1y_1p_2 + rh_1y_1b_2 \text{ since } P \text{ is an ideal of } N , t
\]
Then
\[
-p_3, rp_1y_1p_2, rp_1y_1b_2, rh_1y_1p_2, rh_1y_1b_2 \in P \\
\]
For some \( p_4 \in P \). Thus \( b = p_4 + rh_1y_1b_2 \)
So \( rh_1y_1b_2 = b - p_4 \) hence
\[
= b \in P \subseteq P \subseteq P.
\]

3-Intuitionistic fuzzy bi-ideal with respect to an element of a near ring

In this section we devoted to study fuzzy bi-ideal with respect to an element of a near ring \( N \), we introduce the notion intuitionist fuzzy bi-ideal with respect to an element of the near ring \( N \), and give some properties, theorem about this ideals.

Definition (3.1)
A fuzzy set \( \mu \) of a near ring is called fuzzy bi-ideal with respect to an element of a near ring \( N \) if
1. \( \mu(r(x - y)) \geq \min \{ \mu(rx), \mu(ry) \} \)
   \[ \forall x, y \in N, r \in N. \]
2. \( \mu(r(xyz)) \geq \min \{ \mu(rx), \mu(rz) \} \)
   \[ \forall x, y, z \in N, r \in N. \]
It denoted by \( F-r-bi-ideal \) of \( N \).

Example (3.2)
Consider the near ring \( N= \{0,1,2,3\} \)
With addition and multiplication defined by the following tables.

<table>
<thead>
<tr>
<th>+</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>.</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

The fuzzy subset \( \mu \) of \( N \) which is defined by
\[
\mu(y) = \begin{cases} 
0.9 & \text{if } y = 0,1 \\
0 & \text{otherwise} 
\end{cases}
\]
\( \mu \) is \( F-2-bi-ideal \) of \( N \)

Definition (3.3)
An intuitionistic fuzzy set \( A = (\mu_A, \lambda_A) \) in \( N \) is an intuitionistic fuzzy bi-ideal with respect to an element \( r \) of \( N \) if for all \( x, y, z \in N, r \in N \),
(1) \( \mu_A(r(x - y)) \geq \min \{ \mu_A(rx), \mu_A(ry) \} \)
(2) \( \mu_A(r(xy z)) \geq \min \{ \mu_A(rx), \mu_A(rz) \} \)
(3) \( \lambda_A(r(x - y)) \leq \max \{ \lambda_A(rx), \lambda_A(ry) \} \)
(4) \( \lambda_A(r(xy z)) \leq \max \{ \lambda_A(rx), \lambda_A(rz) \} \)

**Theorem (3.4)**
An intuitionistic fuzzy set \( A = (\mu_A, \lambda_A) \) in \( N \) is an intuitionistic F-r- bi-ideal of \( N \) if and only if the fuzzy set \( \mu_A \) and \( \lambda_A \) are F-r- bi-ideal of \( N \).

**Proof**
If \( A = (\mu_A, \lambda_A) \) is an intuitionistic F-r- bi-ideal of \( N \), then clearly \( \mu_A \) is a F-r- bi-ideal of \( N \), for all \( x, y \in N \),
\[
\lambda_A^c(r(x - y)) = 1 - \lambda_A^c(r(x - y)) 
\]
\[
\geq 1 - \max \{ \lambda_A^c(rx), \lambda_A^c(ry) \} 
\]
\[
= \min \{ 1 - \lambda_A^c(rx), 1 - \lambda_A^c(ry) \} 
\]
\[
= \min \{ \lambda_A^c^c (rx), \lambda_A^c^c (ry) \} , \forall x, y \in N, r \in N, 
\]

\[
\lambda_A^c(r(xy z)) = 1 - \lambda_A^c(r(xy z)) 
\]
\[
\geq 1 - \max \{ \lambda_A^c(rx), \lambda_A^c(rz) \} 
\]
\[
= \min \{ 1 - \lambda_A^c(rx), 1 - \lambda_A^c(rz) \} 
\]
\[
= \min \{ \lambda_A^c^c (rx), \lambda_A^c^c (rz) \} 
\]

thus \( \lambda_A^c \) is a F-r- bi-ideal of \( N \). Conversely that \( \mu_A^c \) and \( \lambda_A^c \) are F-r- bi-ideal of \( N \), then clearly the conditions 1,2 of definition (3.3) are valid. Now for all \( x, y \in N, r \in N \),
\[
1 - \lambda_A^c(r(x - y)) = \lambda_A^c(r(x - y)) 
\]
\[
= \min \{ \lambda_A^c^c (rx), \lambda_A^c^c (ry) \} 
\]
\[
= 1 - \max \{ \lambda_A^c^c (rx), \lambda_A^c^c (ry) \} 
\]

therefore \( \lambda_A^c(r(x - y)) \leq \max \{ \lambda_A^c(rx), \lambda_A^c(ry) \} \) \( \forall x, y, z \in N, r \in N \),
\[
1 - \lambda_A^c(r(xy z)) = \lambda_A^c(r(xy z)) 
\]
\[
\geq \min \{ \lambda_A^c^c (rx), \lambda_A^c^c (rz) \} 
\]
\[
= 1 - \max \{ \lambda_A^c^c (rx), \lambda_A^c^c (rz) \} 
\]

therefore \( \lambda_A^c(r(xy z)) \leq \max \{ \lambda_A^c(rx), \lambda_A^c(rz) \} \)

Thus \( A = (\mu_A, \lambda_A) \) is an intuitionistic F-r- bi-ideal of \( N \).

**Theorem (3.5)**
An intuitionistic fuzzy set \( A = (\mu_A, \lambda_A) \) in \( N \), is an intuitionistic F-r- bi-ideal of \( N \) if and only if \( A = (\mu_A, \mu_A^c) \) and \( A = (\lambda_A^c, \lambda_A) \) are intuitionistic F-r- bi-ideal of \( N \).

**Proof**
If \( A = (\mu_A, \lambda_A) \) is an intuitionistic F-r- bi-ideal of \( N \), then \( \mu_A = (\mu_A^c)^c \) and \( \lambda_A^c \) are F-r- bi-ideal of \( N \), from theorem (3.4). Therefore \( A = (\mu_A, \mu_A^c) \) and \( A = (\lambda_A^c, \lambda_A) \) are intuitionistic F-r- bi-ideal of \( N \).
Conversely if \( A = (\mu_A, \lambda_A) \), \( A = (\lambda_A, \lambda_A) \) are intuitionistic \( F_r \)- bi-ideal of \( N \), then the fuzzy sets \( \mu_A \) and \( \lambda_A \) are \( F_r \)-bi-ideal of \( N \), therefore \( A = (\mu_A, \lambda_A) \) is intuitionistic \( F_r \)-bi-ideal of \( N \).

**Proposition (3.6)**

An intuitionistic fuzzy set \( A = (\mu_A, \lambda_A) \) in \( N \) is a \( F_r \)-bi-ideal of \( N \) if and only if all non-empty set \( \cup (\mu_A, h) \) and \( \cap (\lambda_A, t) \) are \( r \)-bi-ideal of \( N \) for all of \( A \in \text{Im}(\mu_A) \) and \( t \in \text{Im}(\lambda_A) \) respectively .

**Proof**

Suppose that \( A = (\mu_A, \lambda_A) \) is an intuitionistic \( F_r \)-bi-ideal of \( N \), for \( x, y \in \cup (\mu_A, h) \), we have \( \mu_A (r(x - y)) \geq \min \{\mu_A (rx), \mu_A (ry)\} \geq h \)

Therefore \( r(x - y) \in \cup (\mu_A, h) \) let \( x, z \in \cup (\mu_A, h) \) and \( y \in N \).then \( \mu_A (r(xyz)) \geq \min \{\mu_A (rx), \mu_A (rz)\} \geq h \) and so \( r(xyz) \in \cup (\mu_A, h) \) hence \( \cup (\mu_A, h) \) is a \( r \)-bi-ideal of \( N \).

For all \( h \in \text{Im}(\mu_A) \).similarly we can show that \( \cup (\lambda_A, t) \) is also a \( r \)-bi-ideal of \( N \) for all \( t \in \text{Im}(\lambda_A) \).Conversely suppose that \( \cup (\mu_A, h) \) and \( \cup (\lambda_A, t) \) are \( r \)-bi-ideal of \( N \) for all \( h \in \text{Im}(\mu_A) \) and \( t \in \text{Im}(\lambda_A) \) respectively .Suppose that \( x, y \in N \) and \( \mu_A (r(x - y)) \leq \min \{\mu_A (rx), \mu_A (ry)\} \)

Choose \( h \) such that \( \mu_A (r(x - y)) < h < \min \{\mu_A (rx), \mu_A (ry)\} \) Then we get \( x, y \in \cup (\mu_A, h) \) but \( r(x - y) \not\in \cup (\mu_A, h) \) a contradiction. Hence \( \mu_A (r(x - y)) \geq \min \{\mu_A (rx), \mu_A (ry)\} \).

A similar argument shows that \( \mu_A (r(xyz)) \geq \min \{\mu_A (rx), \mu_A (rz)\} \)

For all \( x, y, z \in N \).likewise we can show that

\[
\lambda_A (r(x - y)) \leq \max \{\lambda_A (rx), \lambda_A (ry)\}
\]

\[
\lambda_A (r(xyz)) \leq \max \{\lambda_A (rx), \lambda_A (rz)\}
\]

Hence \( A = (\mu_A, \lambda_A) \) is an intuitionistic \( F_r \)-bi-ideal of \( N \).

**Theorem (3.7)**

A non empty set \( B \) of \( N \) is \( r \)-bi-ideal of \( N \) if and only if \( A = (\chi_B, \chi_B^c) \) is an intuitionistic \( F_r \)-bi-ideal of \( N \).

**Proof**

Straight forward.

**References**