Complex Dynamics in incoherent source with ac-coupled optoelectronic Feedback

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Abstract:
The appearance of Mixed Mode Oscillations (MMOs) and chaotic spiking in a Light Emitting Diode (LED) with optoelectronic feedback theoretically and experimentally have been reported. The transition between periodic and chaotic mixed-mode states has been investigated by varying feedback strength. In incoherent semiconductor chaotically spiking attractors with optoelectronic feedback have been observed to be the result of canard phenomena in three-dimensional phase space (incomplete homoclinic scenarios).

Keywords: Chaos, light-emitting diodes, Mixed Mode Oscillations, optoelectronic feedback.

Introduction
Oscillatory dynamics in chemical, biological, and physical systems often takes the form of complex temporal sequences known as mixed-mode oscillations (MMOs) [1]. The MMOs refers as complex patterns that arise in dynamical systems, in which oscillations with different amplitudes are interspersed. These amplitude regimes differ roughly by an order of magnitude. In each regime, oscillations are created by a different mechanism and their amplitudes may have small variations. These mechanisms govern the transition among regimes [2]. Additionally, MMOs have been observed in laser systems, LEDs and in neurons [1, 3]. In a single LED, R. Meucci et al. (2012) demonstrate numerically and experimentally the occurrence of complex sequences of periodic/chaotic Mixed Mode Oscillations (MMOs) [4]. The crucial element to induce MMOs in optoelectronic devices is the existence of a threshold for light emission. In a LED the main recombination mechanism is the...
spontaneous emission and the emitted light is simply proportional to the current through the junction. However, as in electronic diodes, the current–voltage characteristics of a LED are highly nonlinear and emission/conduction occurs only beyond a threshold voltage [5]. In M.P. Hanias et al. (2011) showed that the LED exposed chaotic behavior even if it works in its operation point, and the obtained simulation results indicate that the proposed circuit can be used to generate chaotic signal, in a light emitting manner, useful in code and decode applications. Hanias demonstrated that the simple externally triggered optoelectronic circuit can be used in order first to generate chaotic voltage signals and then to control the obtained chaotic signals by varying specific circuit parameters [6]. Qualitatively different mechanisms have been proposed to explain the generation of MMOs in other models [7]. These are: break-up of an invariant torus [8] and break-up (loss) of stability of a Shilnikov homoclinic orbit [9, 10] subcritical Hopf-homoclinic bifurcation [7]. Hopfbifurcations in fast-slow systems of ordinary differential equations can be associated with surprising rapid growth of periodic orbits. This process is referred to as candan explosion [11]. However, periodic-chaotic sequences and Farey sequences of MMOs do not necessarily involve a torus or a homoclinic orbit, but can occur also through the canard phenomenon [1]. Canards were first found in a study of the van der Pol system using techniques from nonstandard analysis [12, 13], were first studied in 2D relaxation oscillators [12, 14]. There, the nature of the classical canard phenomenon is the transition from a small amplitude oscillatory state (STO) created in a Hopf bifurcation to a large amplitude relaxation oscillatory state within an exponentially small range of a control parameter. This transition, also called canard explosion, occurs through a sequence of canard cycles which can be asymptotically stable [7]. The dynamics of LEDs can be typically described in terms of two coupled variables (intensity and carrier density) evolving with very different characteristic time scales. The introduction of a third degree of freedom describing the AC-feedback loop, leads to a three-dimensional slow-fast system, displaying complicated bifurcation sequences arising from the multiple time-scale competition between optical intensity, carriers and the feedback nonlinear filter function. A similar scenario has been recently observed in semiconductor lasers with optoelectronic feedback [15, 16].

The dynamical model

The simplest approach is to phenomenologically model the LED as an ideal p-n junction with a uniform recombination region of cross-sectional area S and width ∆. The system dynamics is then determined by three coupled variables, the carrier (electron) density N, the junction applied voltage V_d, and the high-pass-filtered feedback voltage V_f, evolving with very different characteristic time scales.

\[
\dot{N} = -\gamma_{sp}N + \mu N(V_d - V_{bi})/\Delta^2
\]

(1)

\[
CV_d = (V_0 - V_d + f_f(V_f))/R - e\mu N(V_d - V_{bi})/\Delta
\]

(2)

\[
V_f = -\gamma f_f + K\dot{\phi}
\]

(3)

where \(\gamma_{sp}\) is the spontaneous emission rate, \(\mu\) is the carrier mobility, \(V_{bi}\) is a built-in potential, \(C\) is the diode capacitance (here assumed to be voltage independent for simplicity), \(V_0\) is the dc bias voltage, \(R\) is the current-limiting resistor, \(f_f(V_f)\) is the feedback amplifier function, \(e\) is the electron charge, \(\gamma_f\) is a cut off frequency, \(K\) is the photodetector responsivity, and \(\dot{\phi}\) is the photon density, which is assumed to be linearly proportional to the carrier density \(\dot{\phi} = \eta N\), where \(\eta\) is the LED quantum efficiency. Equation (1) indicates that \(N\) in the active layer decreases due to radiative recombination and increases with the forward injection current density \(J = e\mu N(V_0 - V_{bi})/\Delta\). Nonradiative recombination and carrier generation by optical absorption have been neglected since we checked that they do not significantly change the dynamics. Equation (2) is the Kirchhoff law of the circuit (resistor-ideal diode) relating the junction voltage \(V_d\) to the dc applied voltage \(V_0\) [the second term in Eq. (2) is the current flow across the diode \(I = JS\)]. Equation (3) describes the nonlinear feedback loop where the voltage signal coming from the detector \(k\phi\) is high-pass filtered and added to the dc bias through the amplifier function \(f_f(V_f)\). Consider just the solitary LED equations (1) and (2). A finite stationary carrier density, increasing linearly with \(V_0\), is found only when \(V_d > V_{bi} = V_{bi} + \gamma_{sp}K^2/\mu\); otherwise, the only stationary solution is \(N = 0\) and \(V_d = V_0\) (zero current). Accordingly, light emission begins when the applied voltage \(V_0\) exceeds the threshold voltage \(V_{thr}\). By introducing the dimensionless variables \(x = e\mu RSN/\Delta\), \(y = \mu (V_d - V_{bi})/\gamma_{sp}\), and \(z = e\mu RS/\kappa\phi\Delta V_d - x\) and the time scale \(t = \gamma_{sp}t\), so Eqs. (1), 2 and (3) become [1]:

\[
x = x_1*\gamma(y_1 - 1) + y_1
\]

(4)

\[
y = -\gamma((\delta + y_1 - a^*(x_1 + z_1)/(1 + s^*(x_1 + z_1))) + x_1*y_1)
\]

(5)
\[ \dot{z} = -\varepsilon^* (z_1 + x_1) \]  \hspace{1cm} (6)

- \( x \)-the photon density, \( y \)-carrier density, \( z \)-high-pass filtered feedback current
- \( \varepsilon \)-feedback strength, \( \delta \)-bias current.

**Numerical simulations results**

The numerical work is achieved by implementing a dynamical model (i.e. Al-Naimee model) with the use of fourth-order Runge-Kutta integration scheme, with time-step \( dt = 0.01 \). The total simulation time chosen depends strongly on the magnitude of the temporal scales defined by the parameters \( \gamma, \delta \) and \( \varepsilon \).

It is noticed from Figure-1a, that the system periodic self-oscillations or Mixed Mode Oscillations (MMOs) as the feedback strength decreased in which a number of large amplitude oscillations \( L \) is followed by a number of small amplitude oscillations \( S \) to form a complex pattern \( (L_1 S_1 L_2 S_2 \ldots) \) in the time series. In our system \( L = 1 \) But \( S = 1, 2, 3, 4 \) When \( \varepsilon = 0.00122, 0.0012, 0.00108, 0.00092 \) respectively. At \( \varepsilon = 0.00091 \), the system become chaotic. When \( \varepsilon = 0.000216 \) the system is period doubling and it is periodic at \( \varepsilon = 0.0001 \). In these MMOs the ratio of \( L \) to \( S \) amplitudes is 1:1, 1:2, 1:3, 1:4 respectively.

In Figure-1b shows the corresponding FFT for these states where the distribution is decay exponentially where many distinguished frequency could be seen. The small orbits of the attractor in Figure-1c represent the low amplitude oscillations in the time series while the high amplitude spikes have been represented as large orbits. Between the steady state and the chaotic spiking the system passes through a cascade of period doubled and chaotic attractors of small amplitude. This is illustrated by Figure-2, where a bifurcation diagram is computed from our system varying \( \varepsilon \) over a small interval contiguous to the initial Hopf bifurcation [17].

A system transition from one type of behavior to another depending on the value of a set of important parameters like \( \varepsilon \) value. When the \( \varepsilon \) value is changing from 0 to maximum value \( 2.5 \times 10^{-4} \) the amplitude of \( x \) scale in time series divide in three parts. The first one is curve line where \( \varepsilon \) value is starting from \( 1 \times 10^{-5} \) to \( 1.75 \times 10^{-4} \) that called regular oscillation. The second part called period doubling that is starting from \( 1.75 \times 10^{-4} \) to \( 2.3 \times 10^{-4} \). Finally, the chaos behavior region is starting from \( 2.3 \times 10^{-4} \) to \( 2.5 \times 10^{-4} \) as shown in Figure-2.
Figure 1- Numerical simulations of Al-Naimee model, a-Time series at different $\varepsilon$ value but $\gamma=0.01$, $S=0.2$, $a=1$ and $\delta=2.75$. 

$\varepsilon=0.00122$ $L=1$, $S=1$

$\varepsilon=0.0012$ $L=1$, $S=2$

$\varepsilon=0.00108$ $L=1$, $S=3$

$\varepsilon=0.000216$ $L=1$, $S=4$

$\varepsilon=0.0001$ chaotic system

$\varepsilon=0.000216$ period doubling

$\varepsilon=0.0001$ periodic
Figure 1- Numerical simulations of Al-Naimee model, b-The corresponding FFT at different $\varepsilon$ value but $\gamma=0.01$, $S=0.2$, $a=1$ and $\delta=2.75$. 

Note: The image contains graphs showing numerical simulations and corresponding FFT for different $\varepsilon$ values.
Figure 1 - Numerical simulations of Al-Naimee model, c-The corresponding attractor by drawing the relationship between photon density equation (x1) vs. population inversion equation (y1) at different $\varepsilon$ value but $\gamma=0.01$, $S=0.2$, $a=1$ and $\delta=2.75$. 
These mixtures can form periodic sequences following the Farey arithmetic and plotting of a suitably defined winding number $R$ against the bifurcation parameter leads to a devil’s staircase [1] where the winding number can calculated by $R = L / (L+S)$, $L$ is the number of large amplitude oscillations in a single periodic pattern, $S$ is the number of small amplitude oscillations. Devil's staircase is the picturesque used to describe the intricate, often fractal, structure of such staircases [18]. The complete bifurcation diagram corresponding to the mixed-mode wave forms can be represented by plotting the winding number as a function of the control parameter $\varepsilon$ Figure-3. In our system we have $L = 1$ and hence states $1^0$, $1^1$, $1^2$, $1^3$, and $1^4$ have $R$ equal to 1, 1/2, 1/3, 1/4, and 1/5, respectively. The typical sequence of MMOs states is observed as $\varepsilon$ decreased.

**Experimental setup and results**

The experimental setup is illustrated in Figure-4. We consider a closed-loop optical system, consisting of a semiconductor device with AC-coupled nonlinear optoelectronic feedback. The device is consisting of a gallium arsenide infrared emitting diode optically coupled to a monolithic silicon phototransistor detector. The output light is sent to a photodetector producing a current proportional to the optical intensity. The corresponding signal is sent to a variable gain amplifier characterized by a nonlinear transfer function of the form $f(w) = Aw / (1 + sw)$, where $A$ is the amplifier gain and $s$ a saturation coefficient, and then fed back to the injection current of the LED. The feedback strength is determined by the amplifier gain, while its high-pass frequency cutoff can be varied (between 1 Hz
and 100 KHz) by means of a tunable high-pass filter. External signals or control bias can be added to the LED pumping current.

![Image](https://via.placeholder.com/150)

**Figure 4** - Sketch of the experimental setup for LED.

Figure-5 is set of graphs representing the dynamic behavior of the experimental setup when the bias current is fixed at (0.4712) mA. The column (a) in Figure-5 shows the time series when the beginning of the behavior of MMOs appears. The increasing of feedback strength to 0.0046 is lead to MMOs of order 1 as shown in Figure-5a (1). The MMOs of order 1 is appeared at 0.0513 as shown in Figure-5a (2), and the MMOs transition between 1 and 1 at 0.056, as shown Figure-5a (3), and the MMOs of order 1 at 0.0653, as shown Figure-5a (4). Then the MMOs of order 1 at 0.093, as shown in Figure-5a (5), and so on. The column (b) in Figure-5 demonstrates the decay exponentially of the corresponding FFT, and column (c) in Figure-5 demonstrates the corresponding attractor which shows the beginning of the existence of one large phase-space orbit with high amplitude (L), and small phase-space orbit with low amplitude (S) where used the impeded technique to plot it.
Figure 5 - (a) Experimental time series for a single LED with optoelectronic feedback as amplifier gain is increased: (a-1) G=0.0046, (a-2) G=0.0513, (a-3) G=0.056, (a-4) G=0.0653, (a-5) G=0.093.
Figure 5-(b) the corresponding FFT, (1) $G=0.0046$, (2) $G=0.0513$, (3) $G=0.056$, (4) $G=0.0653$, (5) $G=0.093$
The system periodic self-oscillations or (MMOs) as the feedback strength increased and by fixing the bias current. In correspondence to the large pulses, each oscillation period can be decomposed into a sequence of periods of slow motion, near extrema, separated by fast relaxations between them. This behavior (relaxation oscillations) is typical of slow–fast systems. To examine the influence of the gradually increasing in the feedback strength on the output dynamics of the incoherent source (LED) while the DC bias current is kept constant, the bifurcation diagram is sketched for increment values of feedback strength as illustrated in Figure (6).

**Figure 6** -(C)Experimental bifurcation diagram (maxima of photon densities vs. feedback strength variation).
In contrast to the numerical, as the feedback strength is increased, we observe the complete sequence of transitions going from the $1^1$ state to $1^7$, thus reproducing qualitatively the numerical results. However, the detailed bifurcation diagram in terms of the winding number $R$ reveals an extraordinary complexity as shown in Figure-7. The typical sequence of MMOs states that is observed as feedback strength increased is: $1^1 (R = 1/2)$ and $1^2 (R = 1/3)$ and $1^3 (R = 1/4)$ and $1^4 (R = 1/5)$ and $1^5 (R = 1/6)$ and $1^6 (R=1/7)$ finally $1^7 (R=1/8)$. In the transition between these states, random concatenations of the adjacent patterns are observed.

![Bifurcation Diagram](image)

**Figure 7** - Plot of the winding number $R$ as a function $V_n$.

**Conclusion**

We demonstrate numerically and experimentally the appearance of complex periodic mixed-mode oscillations in a light-emitting diode with optoelectronic feedback. Numerically, the transitions from periodic to period doubling, MMOs and chaotic states, by varying feedback strength and fixing bias current have been noticed. The chaotic behavior could be studied in terms of FFT and attractor corresponding to time series. It is noticed that the system periodic self-oscillations or Mixed Mode Oscillations (MMOs) as the feedback strength decreased. Experimentally, Mixed Mode Oscillations (MMOs) as the feedback strength increased have been noticed. There is a difference between numerical and experimental due to the feedback is negative in experimental.

**References**